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# A Matheuristic for the Swap-body Vehicle Routing Problem

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**Abstract** We consider the swap-body vehicle routing problem, an extension of the capacitated Vehicle Routing Problem, in which intermediate locations can be used to change the configuration of a vehicle. Possible actions include a parking operation that decouples the semi-trailer as well as a swapping operation that switches the swap body with another one, which was previously parked at an intermediate location. Successful ideas that combine mathematical programming and heuristics have been recently presented for several vehicle routing problems. In this line, the contribution of this paper is the development of a column generation-based approach, in which a variable neighborhood search heuristic populates the route pool. A detailed numerical analysis is carried out for 138 instances with up to 1,000 customers and at most 100 intermediate locations. Comparisons with existing metaheuristics show that our solution strategy is suitable for solving this problem. It has obtained 42 new best known solutions with a maximal improvement of 18.54% on published benchmark instances.

**Keywords** vehicle routing problem · matheuristic · VeRoLog challenge · swap-body

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## 1 Introduction

This paper proposes a matheuristic for the Swap-Body Vehicle Routing Problem (SB-VRP) as defined in the context of the first Vehicle Routing and Logistics Optimization (VeRoLog) solver challenge 2014 (Heid et al, 2014). In this problem a fleet of trucks as well as combinations of a truck with an attached semi-trailer serves a set of customers $V_c$ from one depot $v_0$. A distinction is made between truck customers $V_c^{Truck}$, train customers $V_c^{Train}$ and mandatory train customers $V_c^{mTrain}$. Due to manoeuvring space, small streets and other constraints, truck customers can solely be visited by a truck. Train customers can either be delivered by a truck pulling a semi-trailer or by a truck alone. Mandatory train customers must be serviced by a train since the delivery quantity of these customers exceeds the capacity $Q$ of one swap-body carried by a truck solely. Thus, it holds that $V_c = V_c^{Truck} \cup V_c^{Train} \cup V_c^{mTrain}$, $V_c^{Truck} \cap V_c^{Train} = \emptyset$, $V_c^{Truck} \cap V_c^{mTrain} = \emptyset$ and $V_c^{Train} \cap V_c^{mTrain} = \emptyset$. Each customer $i$ has a demand $q_i$ and a service time $s_i$.

The distance $d_{ij}$ and the driving time $t_{ij}$ between any two nodes $i$ and $j$ are known. Since the problem is based on a real-world application, the $d_{ij}$ as well as the $t_{ij}$ values are asymmetric and the triangular inequality is not assumed to be satisfied. Furthermore, a maximal driving time $T$ for each driver must be respected, which incorporates the travel times, service times as well as the operation times at the swap locations.

One of the distinguishing features of the SB-VRP is that several actions can be applied at the predefined swap locations $v_s$. This gives rise to complex routes with a main first level trip and one or more second level trips as shown in Table 1 as well as in Figure 1. One major difference between those first and second level trips is the vehicle configuration. On the first level trip a truck is coupled with a semi-trailer. On the second level trip only a truck serves the customers.

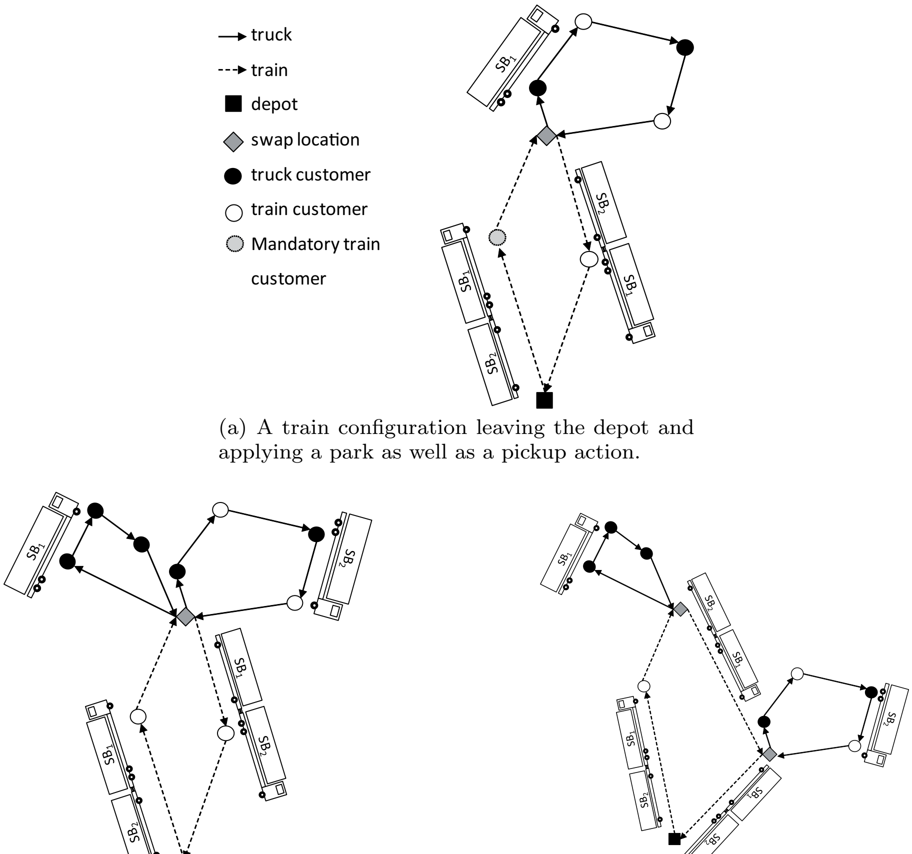
An overview of the actions and the states of the vehicle configuration visiting a swap location is presented in Table 1. Information about the three states is repeated

Table 1: States of the train configuration using the swap location(s). Note that the first position in the vector corresponds to the swap-body the truck carries and the second position to the swap-body the semi-trailer transports.

| Actions | Vehicle state before $v_s$ | Storage of $SB_l$ at $v_s$ | Vehicle state after $v_s$ |
|---|---|---|---|
| Park | $(SB_1, SB_2)$ | $(\emptyset, SB_2)$ | $(SB_1, \emptyset)$ |
| Swap | $(SB_1, \emptyset)$ | $(\emptyset, SB_1)$ | $(SB_2, \emptyset)$ |
| Exchange | $(SB_1, SB_2)$<br>$(SB_2, SB_1)$ | $(\emptyset, SB_1)$<br>$(\emptyset, SB_2)$ | $(SB_2, \emptyset,)$<br>$(SB_1, \emptyset)$ |
| Pickup | $(SB_1, \emptyset)$<br>$(SB_2, \emptyset)$ | $(\emptyset, SB_2)$<br>$(\emptyset, SB_1)$ | $(SB_1, SB_2)$<br>$(SB_2, SB_1)$ |

for each action: 1) the vehicle state before driving to $v_s$, 2) the storage of the swap-body $SB_l$ with $l = 1, 2$ at $v_s$ and 3) the state after $v_s$. The vector encodes for every state the swap-body carried by the truck (first position in the vector) and the swap-body used by the semi-trailer (second position). For example, the train configuration starts with the truck carrying $SB_1$ and $SB_2$ is assigned to the semi-trailer $((SB_1, SB_2))$. Then, the configuration moves to the swap-location $v_s$, where the semi-trailer with $SB_2$ is decoupled $((\emptyset, SB_2))$. We use the notation $\emptyset$ on the first position in the vector to illustrate that the truck utilizing $SB_1$ is currently not at $v_s$. After parking the semi-trailer, only the truck carrying $SB_1$ is moving in the network $((SB_1, \emptyset))$.

A graphical example of a truck pulling a semi-trailer and then parking $SB_2$ at the swap location in order to visit truck customers is illustrated in Figure 1(a). Before continuing to the depot, $SB_2$ must be picked up. An illustration of several actions, such as park, swap and pickup, is presented in Figure 1(b). Figure 1(c) shows a park and a pickup action at the first swap location, which is followed by an exchange operation at a different swap location. This action is necessary since the capacity of $SB_1$ is not sufficient to deliver the two train/truck customers. For the described actions operating times have to be respected. The first action is always parking $SB_2$ since it is not as time-consuming as an exchange action (see also Miranda-Bront et al (2017)).




(a) A train configuration leaving the depot and applying a park as well as a pickup action.

(b) Using a park, a swap as well as a pickup action.

(c) Applying a park and an exchange action.

Fig. 1: Different vehicle states when a train configuration leaves the depot

The problem is to decide on the following aspects:

– what kind of vehicle configurations are leaving the depot (tactical fleet management),
– which swap-locations should be selected,

– what kind of actions should be applied at the swap location and
– in which sequence the customers are delivered

in order to minimize the total costs. The objective function minimizes the sum of the following cost components:

$$\min \sum_{t \in T} C_F^T \times u_t^T + \sum_{s \in S} C_F^S \times u_s^S + \sum_{i,j \in V_c, t \in T} C_D^T \times d_{ij} \times y_{i,j,t}^T + \sum_{i,j \in V_c, s \in S} C_D^S \times d_{ij} \times y_{i,j,s}^S + C_T \times \sum_{i,j \in V_c \cup V_s, t \in T} t_{ij} \times y_{i,j,t}^T + C_T \times \sum_{i \in C} s_i + C_T \times \sum_{i \in V_s, k \in K, s \in S, t \in T, l \in O} \rho_l \times o_{i,k,s,t}^l \tag{1}$$

The first two terms illustrate the occurrence of fixed costs for trucks as well as semi-trailers, respectively. The notation $u_t^T/u_s^S$ indicates whether a truck or a semi-trailer is used and $C_F^T$, respectively $C_F^S$, represents the fixed cost for the truck or the semi-trailer. Distance costs for the trucks and the semi-trailers, respectively are depicted in the next two terms. Thereby, $y_{i,j,t}^T/y_{i,j,s}^S$ is 1 iff the arc is traversed by the truck/semi-trailer and $C_D^T/C_D^S$ gives the cost factor in monetary units per distance. The following terms correspond to the total driving time, the time $s_i$ for servicing the customers as well as the operating times for the actions at the swap locations. $C_T$ expresses the cost for the driver, $\rho_l$ details the time for the selected action and $o_{i,k,s,t}^l$ is 1 iff truck $t$ and semi-trailer $s$ perform operation $l$ at the $k$-th node in the route, i. e. swap location $i$ (Huber and Geiger, 2017; Todosijević et al, 2016).

Our contribution is the presentation of a column generation-based matheuristic, which combines a variable neighborhood search with a set partitioning formulation, leading to improved solutions for 42 benchmark instances. Some of the instances can be greatly improved with a maximal improvement of 18.54%. In particular, we highlight the design choices that enhance the solution quality. For example, the mathematical model strongly relies on a diversified pool of routes resulting in the inclusion of poor solutions. Furthermore, the impact of different strategies for the update of the route pool is analyzed.

The remainder of this paper is structured as follows. Section 2 presents an overview of successful matheuristics for vehicle routing problems. We describe the matheuristic in Section 3, followed by the evaluation of the performance of the proposed method in Section 4. Concluding remarks are given in Section 5.

## 2 Related literature

In total, 27 teams participated in the VeRoLog competition, which is a working group of the Association of the European Operational Research Societies (EURO). The ranking of the finalist teams can be found on the website *http://www.verolog.eu/* and was:

1. Jan Christiaens, Tony Wauters, Túlio A. M. Toffolo, and Sam Van Malderen (Belgium)
2. Juan José Miranda-Bront, Brian Curcio, Federico Pousa, Isabel Méndez-Díaz, Agustín Montero, and Paula Zabala (Argentina)
3. Martin Josef Geiger, and Sandra Huber (Germany)
4. Oliver Lum, Xingyin Wang, Ping Chen, and Bruce Golden (USA)
5. Diego Cattaruzza, Nabil Absi, Dominique Feillet, and Sylvain Housseman (France)
6. Rick van Urk, and A. E. Pérez Rivera (Netherlands)
7. Vedran Muhović, Mirsad Buljubašić, Michel Vasquez, and Haris Gavranović (Bosnia)

8. Raca Todosijević, Saïd Hanafi, Bassem Jarboui, Nenad Mladenović, and Dragan Urošević (France)
9. Mingliang Tao, Yateng Hong, Jun Li, and Meng Wang (China)
10. Vladimir Deineko, Xuan Vinh Doan, and Alexander Tiskin (U. K.)

The winning approach (Toffolo et al, 2018) incorporated automated learning in a stochastic local search. In total 48 neighborhood operators were introduced and the probabilities of selecting each operator were adapted during the execution of the algorithm. New instances for the problem at hand can be downloaded from the website *http://benchmark.gent.cs.kuleuven.be/sbvrp.*

The second ranked approach (Miranda-Bront et al, 2017) presented a cluster-first route-second approach. Two greedy algorithms were developed, which integrated five different operators in the local search phase and included the random removal of customers in the shaking procedure. In addition, a $k$-means clustering algorithm was exploited. On each cluster the proposed greedy randomized adaptive search procedure (GRASP) was carried out independently. Part of the computational time was then used to optimize the merged solution of each cluster.

The work of Huber and Geiger (2014) was ranked third. A variable neighborhood search (VNS) was introduced and an experimental setting was presented in order to determine the importance of the investigated neighborhood operators (Huber and Geiger, 2017). A VNS was developed in Todosijević et al (2016) with nine operators. Absi et al (2015) presented a parallel population based algorithm, where a chromosome represents a giant tour. At this stage, no swap locations were enclosed in the tour. Then, a split procedure was applied to construct feasible solutions. To improve this solution ten moves were involved in the local search phase and a repair procedure was implemented to ensure feasibility of the routes. Moreover, twenty new instances were suggested in Absi et al (2015) and are available under the website *http://vrp-rep.org.*

In Lum et al (2015) a three-phase heuristic was created. The first phase used a simulated annealing algorithm to solve the SB-VRP without the swap locations. Since this solution method tackled the capacitated vehicle routing problem (CVRP), in the second phase a repair procedure was developed to ensure the constraints on the type of the customer. After that, an improvement phase was established using a variable neighborhood descent (VND) procedure incorporating five operators.

In recent years, matheuristics (Parragh and Schmid, 2013; Talbi, 2013) have obtained competitive solutions to many variants of vehicle routing problems and, in particular, to the truck and trailer routing problem (Parragh and Cordeau, 2017), which is similar to the SB-VRP. Matheuristics combine mathematical programming with metaheuristics (Boschetti et al, 2009). Literature surveys recently appeared on matheuristics for rich vehicle routing problems in Archetti and Speranza (2014); Ball (2011); Doerner and Schmid (2010). Different schemes have been defined to classify the approaches in the literature. We focus on the classification of Archetti and Speranza (2014) since the authors combine the findings of the previous studies. They distinguish the following classes:

– Decomposition approaches;
– Improvement heuristics;
– Branch-and-price/column generation-based approaches.

Decomposition approaches divide a problem into smaller subproblems and for each subproblem a solution method is selected. Thereby, some or all of the subproblems are solved with mathematical programming. The idea has received attention since the subproblems are smaller and often can be handled independently (Archetti and Speranza, 2014). Typically, such solution methods are applied on integrated problems, such as the inventory routing problem. This is due to the fact that decisions are made on two different planning levels, namely the determination of delivery quantities as well as the sequencing of the customers (Cordeau et al, 2015). In general, matheuristics belonging to the class of improvement heuristics combine heuristics with the exact solution of a mixed-integer linear programming (MILP) formulation. Often, a heuristic constructs an initial solution, which the MILP aims to improve. The last class incorporates branch-and-price algorithms that have been successfully applied to routing problems. Such approaches make use of a set partitioning formulation, where the columns are associated with each possible route. Since this model has an exponential number of variables, the linear relaxation is solved with column generation. In the branch-and-price/column generation-based approaches the exact method is adapted to speed up the convergence. One example is to terminate the column generation phase prematurely. In this case optimality cannot be proven (Archetti and Speranza, 2014).

Motivated by numerous successful matheuristic contributions since the literature study of Archetti and Speranza (2014), our goal is to highlight recently published matheuristics in the area of vehicle routing problems. In particular, our main focus lies in the class of branch-and-price/column generation-based approaches.

The study of Grangier et al (2017) developed a matheuristic based on large neighborhood search (LNS) for the vehicle routing problem with cross-docking. During the LNS search *pickup legs* as well as *delivery legs* of the local optima are stored in a memory. A pickup leg is the sequence of operations of one vehicle from the depot to the cross-dock and a delivery leg gives the sequence of distributions between the departure at the cross-dock and the return to the depot. The leg pool was utilized in a set partitioning (SP) and matching problem with a third-party solver.

The CVRP and six variants of the VRP are tackled with a hybrid algorithm in Subramanian et al (2013). In the local search phase the authors use a VND with random neighborhood ordering and in the perturbation phase several swap as well as shift moves are applied. A mechanism was introduced that chooses routes for the set partitioning formulation based on the number of customers, the average number of customers per route and on the deviation of the current solution and the best solution so far. Promising values for these measurements were identified through an experimental investigation.

For the VRP with time windows Yıldırım and Çatay (2014) implement a matheuristic that couples ant colony optimization (ACO) with a set partitioning formulation. Furthermore, several elimination rules were proposed in order to prevent duplicate routes. In this context, each route should be assigned a unique identification key. One idea mentioned was the matching of each customer with a prime number and then multiplying the corresponding prime numbers when the customer is in a route. This procedure gives a unique identifier. However, multiplying the prime numbers for larger instances often exceeds the memory storage. Moreover, the authors tested five criteria, that is the total distance, the first customer identifier, the last customer identifier, total number of customers and total tour time. The generated routes are used in the SP to compute an optimal solution. Then, this solution is used in the pheromone update of the ACO.

The pollution-routing problem was addressed with an iterated local search (ILS), a speed optimization algorithm and a SP formulation (Kramer et al, 2015). An initial route was constructed without taking into account different speed levels. Then, the ILS as well as a speed optimization procedure was applied. The SP method was called after the last restart phase if the size of the instance was smaller than a given parameter; otherwise SP was utilized after every restart. In addition, a permanent route pool as well as a temporary pool was established. Routes were derived from the local optima and the pool was cleared after each restart of the hybrid procedure. After each restart phase the best routes were stored in the permanent pool.

An electric fleet size and mix vehicle routing problem with time windows was solved by a hybrid heuristic composed of an ILS and a SP formulation (Penna et al, 2016). This idea was based on the approach in Subramanian et al (2012, 2013). Their memory mechanism favours a set of good quality routes. After each iteration of the multi-start ILS, a SP model is generated. No more details were presented on the identification of good routes for the pool.

A matheuristic was studied for a vehicle routing problem with profit collection in Erdoğan et al (2015). For the metaheuristic component, a tabu search, a LNS as well as a combination of both algorithms were considered. The mathematical programming component was similar to a SP formulation. However, the objective function maximizes the net profit and constraints were included ensuring the specified number for each vehicle type. The size of the set of vehicle routes was set to a maximal number of 1,500 routes with the aim to solve the SP model quickly and to diversify the selection.

To solve the vehicle routing problem with stochastic demand and duration constraints a greedy randomized adaptive search procedure is enhanced with a SP formulation in the study of Mendoza et al (2016). Specifically, two different formulations were used with the following objectives 1) minimization of the total expected duration and 2) minimization of the expected overtime cost. The route pool comprised all routes found in the local optima achieved by the GRASP.

In the study of Parragh and Cordeau (2017) an Adaptive Large Neighborhood Search algorithm was tailored to the truck and trailer routing problem with time windows in order to generate good initial columns for a branch-and-price algorithm. Also, several enhancements to the pricing phase as well as the computation of lower and upper bounds were discussed.

For the asymmetric capacitated vehicle routing problem a hybrid heuristic was tested in the work of Leggieri and Haouari (2016). The solution method combined heuristics with mixed-integer linear programming formulations. In a first step, the graph was reduced by removing unpromising arcs. Also, a pre-assignment of customers to vehicles was utilized. Then, an initial solution was derived by the MILP and a heuristic improvement procedure was invoked to iteratively generate near-optimal solutions.

## 3 Solution method

The proposed matheuristic, called VNS-SP, combines an iterated local search with a set partitioning formulation. An overview of the algorithm is presented in Algorithm 1. Based on the classification in Archetti and Speranza (2014), our idea can be categorized in the branch-and-price/column generation-based approaches. Furthermore, we use a VNS to generate the columns. For the column generation phase a distinction is made between 1) a heuristic that does not take into account the dual information of the restricted master problem and 2) a column generation based on the dual information where only a restricted set of columns is computed (Archetti and Speranza, 2014). Our approach belongs to the first category.

### 3.1 Variable Neighborhood Search

In this section we present the general idea of the VNS-SP heuristic. A highly detailed description of the VNS and the analysis on how to select the neighborhoods in such a

**Algorithm 1** VNS-SP

```
Input: The sequence of neighborhood operators in the VNS: 2EX → INTER → 2OPT → CSL
1: procedure VNS-SP(TimeLimitVNS, TimeLimitSP, NumberCores, SyncTime)
2:     RoutePool ← NULL
3:     s ← NULL
4:     while TimeLimitVNS has not been reached do
5:         for i ← 1, . . . , NumberCores do
6:             if s = NULL then
7:                 s ← GenerateInitialSolution
8:             end if
9:             while time ≤ SyncTime do
10:                s' ← VNS(s)
11:                if f(s) < f(s') then
12:                    s' ← s
13:                    UpdateRoutePool(Routepool, s')
14:                end if
15:                s ← Perturb(s')
16:            end while
17:            if f(s') < f(s*) then
18:                s* ← s'
19:                UpdateRoutePool(Routepool, s*)
20:            end if
21:        end for
22:    end while
23:    while TimeLimitSP has not been reached do
24:        SP_model ← CreateSetPartitioningModel(RoutePool)
25:    end while
26:    return s*
27: end procedure
```

setting is provided in Huber and Geiger (2017). The general procedure of the algorithm has two phases, which are illustrated in Algorithm 1: 1) the VNS (line 1 to 22) and 2) the set-partitioning formulation (line 23 to 25).

A solution of the SB-VRP is represented by a set of train routes as well as truck routes. Each route is composed of segments $S_k$ with $k = 1, \ldots, 4$. However, the number of segments can vary with respect to the type of a route. For example, we present four routes in Figure 2. A train route with four segments is highlighted, which includes the visit of a swap location where three actions must be applied in order to change the configuration, such as park, swap and pickup. It is also possible that in a train route customers are only assigned to three segments. This means that a truck pulling a semi-trailer visits a swap location. After that, the truck configuration is delivering two truck customers before picking up the semi-trailer at the swap location and continuing his route to the depot. Alternatively, a train configuration does not visit any swap location, which means that customers are only assigned to the first segment (bottom left in Figure 2). When a truck configuration is utilized only one segment is used.

An initial solution is constructed by randomly selecting a customer and then assigning it to a segment in a route. Note that a truck or a train configuration is chosen at random. When a customer cannot be inserted in an existing route, a new vehicle configuration is selected. The procedure terminates when all customers are assigned to one segment. After testing the construction procedure, we selected $S_1$, $S_2$, $S_3$ and $S_4$ as a prioritization for the segments.

In the improvement procedure, the VNS utilizes four standard neighbourhood operators in order to improve the objective function value of a solution. An inter-move (*INTER*) operator tries to reposition the selected customer in another route. This move is illustrated in Figure 3, where we selected a train customer of route 1. Then, we try

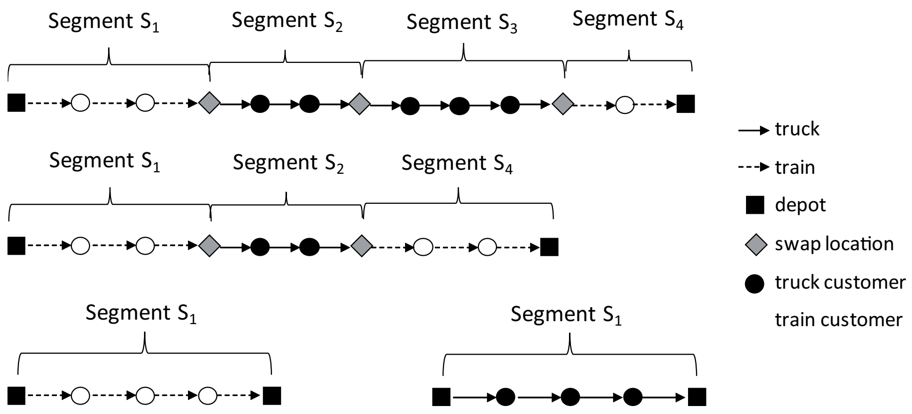



Fig. 2: Representation of a route

to insert this customer in the best position of route 2. A move is accepted when the objective function can be improved. Note that we do not check every route in the current alternative since routes might be far away from each other. Thus, we only check routes which are close to the selected route. This seems to be beneficial since the search space is reduced. The two-inter-exchange (*2EX*) operator swaps the assignment of two selected customers in different routes. A detailed illustration is given in Figure 4, where a train customer in route 1 and a truck customer in route 2 are selected. Every position in $S_2$ of route 1 is tested for the truck customer and every position in $S_1$ is checked for the train customer. A single tour is improved by replacing two arcs with two other arcs (*2OPT*) and the *CSL* operator tests the utilization of different swap locations. The operators are applied in the following sequence: 2EX, INTER, 2OPT and CSL (Huber and Geiger, 2017). In the work of Huber and Geiger (2017) a detailed analysis was provided, which showed that this sequence seemed promising for the majority of the instances considered. Every time the algorithm is stuck in a local optimum, a perturbation move is applied. Thereby, two complete routes are selected at random and removed from the current alternative. Then, these customers are assigned to existing routes. When this is not feasible, a new vehicle configuration is initialized.

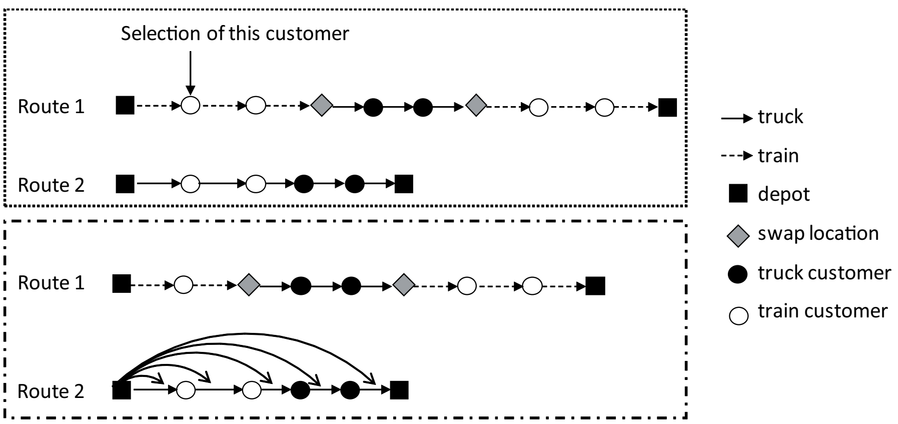



Fig. 3: Inter-move operator

Throughout the heuristic search procedure, a memory $\Omega$ that stores different feasible routes is used. In order to check if a route should be added, every route is labeled with an identifier (e. g. the sum of the identification numbers of the customers in the tour). This is advantageous since the procedure only compares routes with the same identifier (see Section 3.3 for a detailed description).

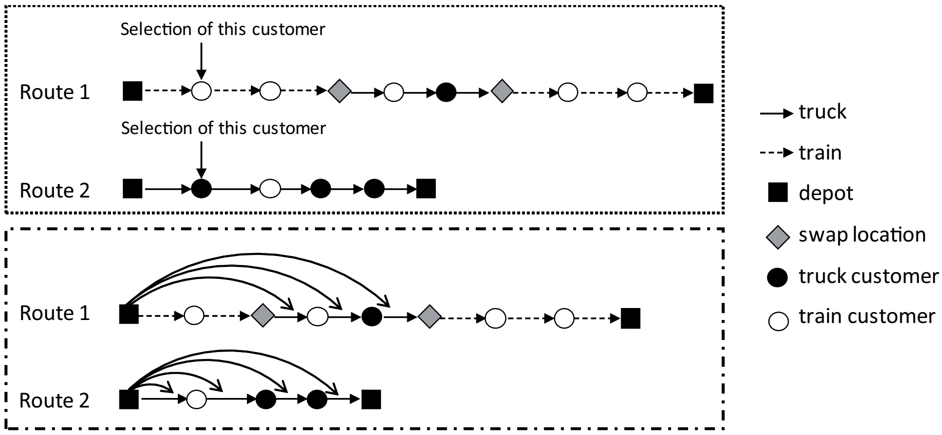



Fig. 4: Two-inter-exchange operator

### 3.2 Set partitioning approach

In the first phase, solutions are discarded based on the cost values compared with the best solution so far. So it might happen that these rejected solutions contain bad truck/train routes as well as good routes, which are also removed. To address this issue, the second phase solves the following set-partitioning formulation:

$$\min \sum_{w \in \Omega} c_w y_w \tag{2}$$

subject to

$$\sum_{w \in \Omega} b_{iw} y_w = 1 \quad \forall i \in V \tag{3}$$

$$y_w \in \{0, 1\} \quad \forall w \in \Omega. \tag{4}$$

For each route $w \in \Omega$, let $c_w$ be the cost of the route. As mentioned above, the cost of the route depends on traveled distance and on whether the truck is moving alone or towing the trailer. The variable $y_w$ takes value 1 if and only if route $w$ is used in the solution. The constant $b_{iw}$ represents the number of times vertex $i \in V$ is traversed by $w$. The objective function in equation (2) minimizes the total cost of the selected routes. Constraints (3) ensure that every customer is visited exactly once. No constraints is included for the swap location since no limits are imposed on the number of visits at the swap location. It is important to mention that we provide the best VNS solution of the first phase for the set partitioning formulation. When a warm-start is not provided it occurs that a better solution cannot be computed within the given time limit. This idea is similar to Grangier et al (2017), where a warm-start is provided to save time.

### 3.3 Pool of routes

Important design choices for the pool of routes should be introduced (Subramanian et al, 2013; Wang et al, 2014). One of the design choices for a unified matheuristic is to take care of the tractability of the MIP model and the improvement potential (Subramanian et al, 2013). During the execution time of 600 seconds several billions of routing plans can be investigated. During the search it should be ensured that only different routes are added to the route pool for the set partitioning formulation. Using different routes is mainly due to the fact that memory issues might occur for larger instances. In addition, running time can be saved during the pre-processing of the MIP solver CPLEX when identical routes are avoided in the VNS phase.

Only feasible routes are included in the pool. Every time a local optimum is computed the pool should be updated. Further experiments were carried out, which showed that it is crucial for high-quality solutions to include further routes. Thereby, we use two strategies: the master of the master-slave principle includes solutions that are within a 10% threshold of the current best solution. Also, solutions of the slaves are kept in the pool to enhance the route diversification.

In order to update the pool, three different route identifiers were tested: the sum of the identification numbers (IDs) of the customers, the ID of the first customer as well as the total costs of one route (rounded to the nearest integer). As an example, we show in Figures 5–7 typical results of the different identifiers are illustrated for the *large2 normal* instance. Comparing the results indicates that the identifier had an influence on the number of keys and the maximum number of different routes. Applying the sum of the customer IDs resulted in the minimal maximum number of different routes (see Figure 5). However, the maximum number of keys was high. The values for the other two identifiers were similar with respect to the number of keys, but the maximum number of different routes with 572 was higher for the ID of the first customer. For further experiments the identifier with the sum of the customer IDs was utilized due to the low number of different routes for each key.

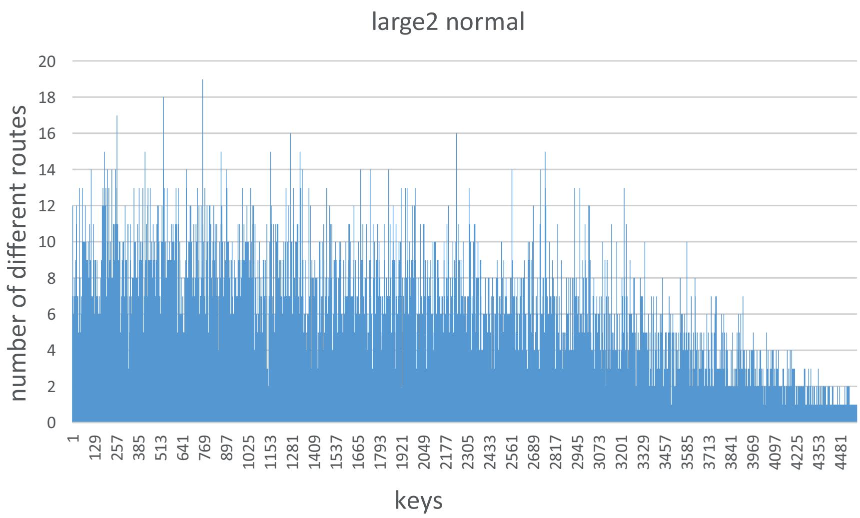



Fig. 5: Identification of the route by the sum of the customer IDs in the route; Number of keys: 4579; Maximum number of different routes: 19

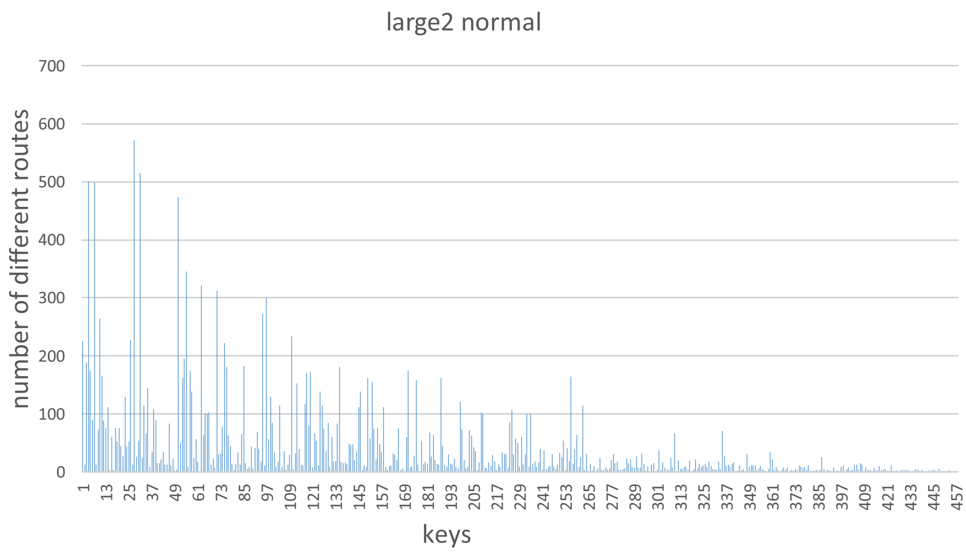



Fig. 6: Identification of the route by the ID of the first customer in the route; Number of keys: 458; Maximum number of different routes: 572

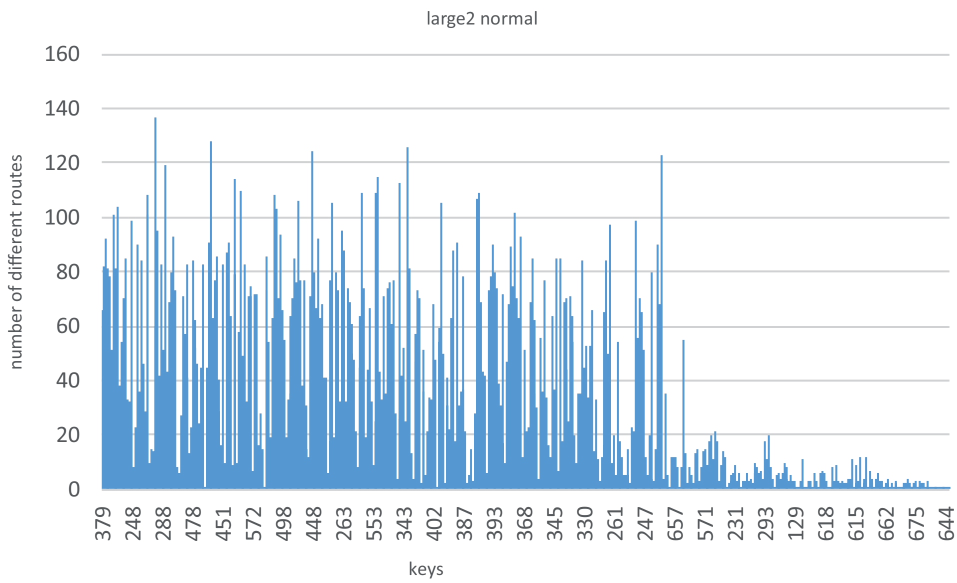



Fig. 7: Identification of the route by the total costs of the route rounded to integer; Number of keys: 535; Maximum number of different routes: 137

## 4 Experimental investigation

The solution approach was coded in Microsoft Visual Basic.NET and CPLEX 12.6.3 was used as a MIP solver. All tests have been carried out on an Intel Xeon X5650 CPU running at 2.66 GHz with 24 GB RAM. The initial experiments were conducted with the VeRoLog challenge rules: the running time was limited to 600 seconds per instance with four cores. However, several experiments were also conducted with a computational time of 1,200 seconds since e. g. the experimental study of Absi et al (2015) and Toffolo et al (2018) included such experiments (see Tables 8, 11 and 14). Best and average results are presented for ten algorithmic executions.

### 4.1 Benchmark instances

The challenge organizers provided test instances for the competition in order to evaluate the performance of the submitted executables. After the challenge, the study of Absi et al (2015) introduced another set of instances in line with the characteristics introduced by the organizers. An additional set of test data was introduced by Toffolo et al (2018) that have the following main differences:

- No maximal time for the driver;
- No travel times;
- No service time operations at the customers;
- No assigned times to the operations at the swap location;
- No fixed costs for the truck and the semi-trailer or driver costs in the objective function;
- Symmetric distances are introduced.

The instances of Absi et al (2015) and Heid et al (2014) can be downloaded from the VRP repository *http://vrp-rep.org*. A description of the characteristics is shown in Tables 2 and 3. In total, 18 instances are provided that vary with respect to the number

Table 2: The instances proposed by the EURO Working Group on Vehicle Routing and Logistics Optimization as well as PTV Group

| # | Instance name | # swap locations | # truck customers | # train customers | # mandatory train customer | released |
|---|---|---|---|---|---|---|
| 1 | small normal | 20 | 15 | 41 | 1 | |
| 2 | small all without tr. | 20 | 57 | 0 | 0 | Feb 1, 2014 |
| 3 | small all with tr. | 20 | 0 | 57 | 0 | |
| 4 | medium normal | 41 | 20 | 186 | 0 | |
| 5 | medium all without tr. | 41 | 206 | 0 | 0 | Feb 1, 2014 |
| 6 | medium all with tr. | 41 | 0 | 206 | 0 | |
| 7 | large1 normal | 99 | 50 | 498 | 0 | |
| 8 | large1 all without tr. | 99 | 548 | 0 | 0 | Feb 1, 2014 |
| 9 | large1 all with tr. | 99 | 0 | 548 | 0 | |
| 10 | large2 normal | 101 | 50 | 500 | 0 | |
| 11 | large2 all without tr. | 101 | 550 | 0 | 0 | May 1, 2014 |
| 12 | large2 all with tr. | 101 | 0 | 550 | 0 | |
| 13 | final random normal | 102 | 50 | 499 | 0 | |
| 14 | final random all without tr. | 102 | 549 | 0 | 0 | July 1, 2014 |
| 15 | final random all with tr. | 102 | 0 | 549 | 0 | |
| 16 | final normal | 102 | 50 | 499 | 0 | |
| 17 | final all without tr. | 102 | 549 | 0 | 0 | July 1, 2014 |
| 18 | final all with tr. | 102 | 0 | 549 | 0 | |

of swap locations and the number of truck, train and mandatory train customers. The new instances of Absi et al (2015) modified the given data sets of the organizers. The type of modification is detailed in the second column of Table 3. However, it is ensured that every modification respects the given constraints. For example the swap body capacity must still be fulfilled for each type of customer when the demand is multiplied by two.

Table 3: Characteristics of the new instances (Absi et al, 2015)

| Class | Modification | Size | #customers |
|---|---|---|---|
| Class 1 | Driving time is multiplied by two. | large1 | 548 |
| | | large2 | 550 |
| | | medium | 206 |
| | | small | 57 |
| Class 2 | Maximal time is divided by two. | large1 | 520 |
| | | large2 | 481 |
| | | medium | 199 |
| | | small | 39 |
| Class 3 | The demand is divided by two. | large1 | 548 |
| | | large2 | 550 |
| | | medium | 206 |
| | | small | 57 |
| Class 4 | The demand is multiplied by two. | large1 | 548 |
| | | large2 | 550 |
| | | medium | 203 |
| | | small | 56 |
| Class 5 | The maximal driving time and the demand of each customer is divided by two. | large1 | 520 |
| | | large2 | 481 |
| | | medium | 199 |
| | | small | 39 |

One hundred new instances have been proposed by Toffolo et al (2018), which are modified instances of the CVRP. An overview of the description of the CVRP instance generator can be found in the work of Uchoa et al (2017). An overview of the new SB-VRP instances is given in Table 4. The name of the instance has always the same structure: the first numerical value corresponds to the number of customers plus one depot and the second value coincides with the number of swap locations. The third column, $\#V_c$, reports the number of customers and the next column presents the position of the depot. The position can be random ($R$), at the corner of the grid ($E$) or in the center of the grid ($C$). Note that the depots, customers and swap locations are on a $[0, 1000] \times [0, 1000]$ grid. The customers can be randomly positioned ($R$) on the grid or clustered (*C(S)* with $S$ being the number of seed customers). Another case is *RC(S)*, where half of the customers are clustered and the remaining customers are randomly assigned. For the demand distributions seven cases are possible:

- U: Unitary demands (value 1) for every customer;
- (1–10): Demands drawn from a uniform distribution $U[1, 10]$;
- (5–10): Demands in $U[5, 10]$;
- (1–100): Demands in $U[1, 100]$;
- (50–100): Demands in $U[50, 100]$;
- Q: Demands are taken in $U[1, 50]$ in an even quadrant and demands are taken in $U[51, 100]$ otherwise. The quadrants are based on point $(500, 500)$;
- SL: 70% to 95% of the demand values are based on $U[1, 10]$. The rest of the customers have a demand value from $U[50, 100]$ (Uchoa et al, 2017).

The capacity of each swap-body is given in column $Q$. This is followed by the number of swap locations in column $\#V_s$. Next, the distance cost of a train per unit $c$ is indicated. The distance for a truck is always set to 1.0 (Toffolo et al, 2018). The last column $\#V_c^{Train}$ gives the number of customers are given that can be visited by a truck- and a train-configuration.

Table 4: Description of the Toffolo et al (2018) instances

| # | Instance | Characteristics | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $V_c$ | Depot | Customers | Demand | Q | $\#V_s$ | c | $\#V_c^{Train}$ |
| 1 | n101-s100 | 100 | R | RC(7) | 1-100 | 206 | 100 | 1.2 | 4 |
| 2 | n106-s20 | 105 | E | C(3) | 50-100 | 600 | 20 | 1.6 | 0 |
| 3 | n110-s4 | 109 | C | R | 5-10 | 66 | 4 | 1.4 | 52 |
| 4 | n115-s100 | 114 | C | R | SL | 169 | 100 | 1.4 | 102 |
| 5 | n120-s4 | 119 | E | RC(8) | U | 21 | 4 | 1.6 | 119 |
| 6 | n125-s20 | 124 | R | C(5) | Q | 188 | 20 | 1.2 | 107 |
| 7 | n129-s4 | 128 | E | RC(8) | 1-10 | 39 | 4 | 1.6 | 0 |
| 8 | n134-s100 | 133 | R | C(4) | Q | 643 | 100 | 1.4 | 11 |
| 9 | n139-s20 | 138 | C | R | 5-10 | 106 | 20 | 1.2 | 138 |
| 10 | n143-s100 | 142 | E | R | 1-100 | 1190 | 100 | 1.4 | 61 |
| 11 | n148-s4 | 147 | R | RC(7) | 1-10 | 18 | 4 | 1.6 | 18 |
| 12 | n153-s20 | 152 | C | C(3) | 1-100 | 144 | 20 | 1.2 | 152 |
| 13 | n157-s20 | 156 | R | C(3) | 1-1 | 12 | 20 | 1.6 | 142 |
| 14 | n162-s100 | 161 | C | RC(8) | 50-100 | 1174 | 100 | 1.4 | 82 |
| 15 | n167-s4 | 166 | E | R | 5-10 | 133 | 4 | 1.2 | 0 |
| 16 | n172-s100 | 171 | C | RC(5) | Q | 161 | 100 | 1.2 | 171 |
| 17 | n176-s4 | 175 | E | R | SL | 142 | 4 | 1.6 | 22 |
| 18 | n181-s20 | 180 | R | C(6) | U | 8 | 20 | 1.4 | 81 |
| 19 | n186-s4 | 185 | R | R | 50-100 | 974 | 4 | 1.2 | 0 |
| 20 | n190-s100 | 189 | E | C(3) | 1-10 | 138 | 100 | 1.6 | 173 |
| 21 | n195-s20 | 194 | C | RC(5) | 1-100 | 181 | 20 | 1.4 | 167 |
| 22 | n200-s100 | 199 | R | C(8) | Q | 402 | 100 | 1.6 | 0 |
| 23 | n204-s4 | 203 | C | RC(6) | 50-100 | 836 | 4 | 1.2 | 101 |
| 24 | n209-s20 | 208 | E | R | 5-10 | 101 | 20 | 1.4 | 208 |
| 25 | n214-s100 | 213 | C | C(4) | 1-100 | 944 | 100 | 1.6 | 23 |
| 26 | n219-s4 | 218 | E | R | U | 3 | 4 | 1.2 | 17 |
| 27 | n223-s20 | 222 | R | RC(5) | 1-10 | 37 | 20 | 1.4 | 105 |
| 28 | n228-s20 | 227 | R | C(8) | SL | 154 | 20 | 1.2 | 0 |
| 29 | n233-s4 | 232 | C | RC(7) | Q | 631 | 4 | 1.4 | 212 |
| 30 | n237-s100 | 236 | E | R | U | 18 | 100 | 1.6 | 236 |
| 31 | n242-s20 | 241 | E | R | 1-10 | 28 | 20 | 1.2 | 0 |
| 32 | n247-s100 | 246 | C | C(4) | SL | 134 | 100 | 1.4 | 215 |
| 33 | n251-s4 | 250 | R | RC(3) | 5-10 | 69 | 4 | 1.6 | 29 |
| 34 | n256-s4 | 255 | C | C(8) | 50-100 | 1225 | 4 | 1.6 | 121 |
| 35 | n261-s100 | 260 | E | R | 1-100 | 1081 | 100 | 1.2 | 260 |
| 36 | n266-s20 | 265 | R | RC(6) | 5-10 | 35 | 20 | 1.4 | 0 |
| 37 | n270-s20 | 269 | C | RC(5) | 50-100 | 585 | 20 | 1.2 | 133 |
| 38 | n275-s4 | 274 | R | C(3) | U | 10 | 4 | 1.4 | 274 |
| 39 | n280-s100 | 279 | E | R | SL | 192 | 100 | 1.6 | 27 |
| 40 | n284-s100 | 283 | R | C(8) | 1-10 | 109 | 100 | 1.6 | 261 |
| 41 | n289-s20 | 288 | E | RC(7) | Q | 267 | 20 | 1.4 | 288 |
| 42 | n294-s4 | 293 | C | R | 1-99 | 285 | 4 | 1.2 | 33 |
| 43 | n298-s100 | 297 | R | R | 1-10 | 55 | 100 | 1.4 | 0 |
| 44 | n303-s4 | 302 | C | C(8) | 1-100 | 794 | 4 | 1.6 | 278 |
| 45 | n308-s20 | 307 | E | RC(6) | SL | 246 | 20 | 1.2 | 140 |
| 46 | n313-s4 | 312 | R | RC(3) | Q | 248 | 4 | 1.2 | 154 |
| 47 | n317-s100 | 316 | E | C(4) | U | 6 | 100 | 1.4 | 316 |
| 48 | n322-s20 | 321 | C | R | 50-100 | 868 | 20 | 1.6 | 286 |
| 49 | n327-s4 | 326 | R | RC(7) | 5-10 | 128 | 4 | 1.2 | 0 |
| 50 | n331-s100 | 330 | E | R | U | 23 | 100 | 1.4 | 32 |
| 51 | n336-s20 | 335 | E | R | Q | 203 | 20 | 1.6 | 159 |
| 52 | n344-s4 | 343 | C | RC(7) | 5-10 | 61 | 4 | 1.2 | 27 |
| 53 | n351-s20 | 350 | C | C(3) | 1-100 | 436 | 20 | 1.6 | 0 |
| 54 | n359-s100 | 358 | E | RC(7) | 1-10 | 68 | 100 | 1.4 | 358 |
| 55 | n367-s20 | 366 | R | C(4) | SL | 218 | 20 | 1.6 | 329 |
| 56 | n376-s100 | 375 | E | R | U | 4 | 100 | 1.4 | 344 |
| 57 | n384-s4 | 383 | R | R | 50-100 | 564 | 4 | 1.2 | 383 |
| 58 | n393-s100 | 392 | C | RC(5) | 5-10 | 78 | 100 | 1.6 | 0 |
| 59 | n401-s20 | 400 | E | C(6) | Q | 745 | 20 | 1.4 | 37 |
| 60 | n411-s4 | 410 | R | C(5) | SL | 216 | 4 | 1.2 | 209 |
| 61 | n420-s4 | 419 | C | RC(3) | 1-10 | 18 | 4 | 1.2 | 198 |
| 62 | n429-s100 | 428 | R | R | 50-100 | 536 | 100 | 1.4 | 37 |
| 63 | n439-s20 | 438 | C | RC(8) | U | 12 | 20 | 1.6 | 438 |
| 64 | n449-s100 | 448 | E | R | 1-100 | 777 | 100 | 1.2 | 407 |
| 65 | n459-s20 | 458 | C | C(4) | Q | 1106 | 20 | 1.4 | 0 |
| 66 | n469-s4 | 468 | E | R | 50-100 | 256 | 4 | 1.6 | 421 |
| 67 | n480-s4 | 479 | R | C(8) | 5-10 | 52 | 4 | 1.6 | 229 |
| 68 | n491-s20 | 490 | R | RC(6) | 1-100 | 428 | 20 | 1.2 | 490 |
| 69 | n502-s100 | 501 | E | C(3) | U | 13 | 100 | 1.4 | 0 |
| 70 | n513-s100 | 512 | C | RC(4) | 1-10 | 142 | 100 | 1.6 | 47 |
| 71 | n524-s4 | 523 | R | R | SL | 125 | 4 | 1.4 | 268 |
| 72 | n536-s20 | 535 | C | C(7) | Q | 371 | 20 | 1.2 | 58 |
| 73 | n548-s100 | 547 | E | R | U | 11 | 100 | 1.2 | 494 |
| 74 | n561-s4 | 560 | C | RC(7) | 1-10 | 74 | 4 | 1.6 | 0 |
| 75 | n573-s20 | 572 | E | C(3) | SL | 210 | 20 | 1.4 | 572 |
| 76 | n586-s20 | 585 | R | RC(4) | 5-10 | 28 | 20 | 1.2 | 65 |
| 77 | n599-s4 | 598 | R | R | 50-100 | 487 | 4 | 1.6 | 598 |
| 78 | n613-s100 | 612 | C | R | 1-100 | 523 | 100 | 1.4 | 0 |
| 79 | n627-s4 | 626 | E | C(5) | 5-10 | 110 | 4 | 1.6 | 549 |
| 80 | n641-s20 | 640 | E | RC(8) | 50-100 | 1381 | 20 | 1.4 | 335 |

Table 4: Description of the Toffolo et al (2018) instances

| # | Instance | Characteristics | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $V_c$ | Depot | Customers | Demand | Q | $\#V_s$ | c | $\#V_c^{Train}$ |
| 81 | n655-s100 | 654 | C | C(4) | U | 5 | 100 | 1.2 | 347 |
| 82 | n670-s4 | 669 | R | R | SL | 129 | 4 | 1.2 | 592 |
| 83 | n685-s20 | 684 | C | RC(6) | Q | 408 | 20 | 1.6 | 684 |
| 84 | n701-s100 | 700 | E | RC(7) | 1-10 | 87 | 100 | 1.4 | 79 |
| 85 | n716-s20 | 715 | R | C(3) | 1-100 | 1007 | 20 | 1.4 | 0 |
| 86 | n733-s4 | 732 | C | R | 1-10 | 25 | 4 | 1.6 | 71 |
| 87 | n749-s100 | 748 | R | C(8) | 1-100 | 396 | 100 | 1.2 | 664 |
| 88 | n766-s100 | 765 | E | RC(7) | SL | 166 | 100 | 1.6 | 0 |
| 89 | n783-s4 | 782 | R | R | Q | 832 | 4 | 1.4 | 782 |
| 90 | n801-s20 | 800 | E | R | U | 20 | 20 | 1.2 | 401 |
| 91 | n819-s100 | 818 | C | C(6) | 50-100 | 358 | 100 | 1.4 | 427 |
| 92 | n837-s4 | 836 | R | RC(7) | 5-10 | 44 | 4 | 1.2 | 759 |
| 93 | n856-s20 | 855 | C | RC(3) | U | 9 | 20 | 1.6 | 0 |
| 94 | n876-s20 | 875 | E | C(5) | 1-100 | 764 | 20 | 1.6 | 100 |
| 95 | n895-s4 | 894 | R | R | 50-100 | 1816 | 4 | 1.2 | 894 |
| 96 | n916-s100 | 915 | E | RC(6) | 5-10 | 33 | 100 | 1.4 | 915 |
| 97 | n936-s4 | 935 | C | R | SL | 138 | 4 | 1.2 | 0 |
| 98 | n957-s100 | 956 | R | RC(4) | U | 11 | 100 | 1.4 | 92 |
| 99 | n979-s20 | 978 | E | C(6) | Q | 998 | 20 | 1.6 | 483 |
| 100 | n1001-s4 | 1000 | R | R | 1-10 | 131 | 4 | 1.6 | 894 |

## 4.2 Impact of the set partitioning formulation

With the goal to illustrate the benefits of our matheuristic, we first apply only the VNS before analyzing the allocation of scarce computational time to the two phases.

Table 5 presents the results found by applying only the VNS on the set of instances suggested in Toffolo et al (2018) and compares them with the numerical values given in Toffolo et al (2018) (T18). The VNS was capable of improving the best solutions in 27 cases. The average gap between the best known solution produced by the VNS and the best known solution so far was 1.23% (see Equation 5).

Table 5: VNS results for the instances proposed in Toffolo et al (2018)

| # | Instance | T18 | | VNS | | Gap Best |
|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol | Best Sol. | Avg. Sol. | |
| 1 | n101-s100 | **22,494.80** | 22,540.70 | 22,517.60 | 22,589.87 | 0.10 |
| 2 | n106-s20 | **23,615.40** | 23,661.40 | 23,649.00 | 23,729.39 | 0.14 |
| 3 | n110-s4 | 15,891.60 | 15,920.80 | **14,911.00** | **14,992.89** | -6.17 |
| 4 | n115-s100 | **12,191.00** | 12,303.00 | 12,558.80 | 12,608.61 | 3.02 |
| 5 | n120-s4 | 13,484.20 | 13,503.40 | **13,397.00** | 13,499.24 | -0.65 |
| 6 | n125-s20 | **36,461.60** | 36,515.20 | 36,580.20 | 36,943.34 | 0.33 |
| 7 | n129-s4 | 30,386.60 | 30,469.40 | **28,517.60** | **28,701.60** | -6.15 |
| 8 | n134-s100 | **9,896.60** | 9,960.20 | 10,106.80 | 10,275.73 | 2.12 |
| 9 | n139-s20 | **12,647.60** | 12,679.30 | 12,676.80 | 12,847.68 | 0.23 |
| 10 | n143-s100 | **14,736.40** | 14,824.00 | 15,474.60 | 16,026.74 | 5.01 |
| 11 | n148-s4 | 44,096.80 | 45,449.00 | **42,127.20** | **42,353.54** | -4.47 |
| 12 | n153-s20 | **14,770.80** | 14,816.00 | 14,805.20 | 14,854.64 | 0.23 |
| 13 | n157-s20 | **15,426.80** | 15,477.30 | 15,794.20 | 15,879.49 | 2.38 |
| 14 | n162-s100 | **13,719.40** | 13,808.00 | 13,976.80 | 14,058.58 | 1.88 |
| 15 | n167-s4 | 19,767.20 | 19,815.60 | **19,537.00** | 19,768.78 | -1.16 |
| 16 | n172-s100 | **30,024.00** | 30,032.50 | 30,032.40 | 30,180.93 | 0.03 |
| 17 | n176-s4 | 46,707.60 | 49,130.30 | **45,599.80** | **45,983.64** | -2.37 |
| 18 | n181-s20 | **22,091.00** | 22,143.20 | 22,580.00 | 22,883.67 | 2.21 |
| 19 | n186-s4 | 25,553.40 | 25,651.50 | **22,794.00** | **23,050.07** | -10.80 |
| 20 | n190-s100 | **15,359.40** | 15,463.50 | 16,520.00 | 16,885.48 | 7.56 |
| 21 | n195-s20 | 37,695.80 | 37,900.70 | **37,473.40** | 37,932.05 | -0.59 |
| 22 | n200-s100 | **50,592.40** | 50,723.50 | 51,335.00 | 51,518.14 | 1.47 |
| 23 | n204-s4 | 20,068.60 | 20,133.10 | **19,108.60** | **19,294.53** | -4.78 |
| 24 | n209-s20 | **26,658.80** | 26,738.30 | 27,181.00 | 27,574.67 | 1.96 |
| 25 | n214-s100 | **10,758.00** | 10,822.30 | 11,091.40 | 11,386.91 | 3.10 |
| 26 | n219-s4 | 100,729.00 | 101,295.40 | **99,089.20** | **99,307.86** | -1.63 |
| 27 | n223-s20 | **36,459.00** | 36,704.40 | 37,188.20 | 37,622.68 | 2.00 |
| 28 | n228-s20 | 22,431.20 | 22,649.60 | **22,422.60** | 22,731.92 | -0.04 |
| 29 | n233-s4 | 18,954.60 | 19,258.70 | **18,697.00** | 18,956.99 | -1.36 |
| 30 | n237-s100 | **25,234.20** | 25,454.60 | 27,082.80 | 27,733.35 | 7.33 |
| 31 | n242-s20 | **62,171.20** | 62,482.20 | 62,734.60 | 63,209.07 | 0.91 |
| 32 | n247-s100 | **28,058.60** | 28,221.80 | 28,885.00 | 29,267.74 | 2.95 |
| 33 | n251-s4 | **37,218.60** | 41,014.90 | 37,634.80 | 37,840.27 | 1.12 |
| 34 | n256-s4 | 21,687.80 | 22,023.10 | **19,017.00** | **19,304.18** | -12.31 |
| 35 | n261-s100 | **20,986.60** | 21,136.90 | 22,198.40 | 22,840.33 | 5.77 |
| 36 | n266-s20 | **63,079.20** | 63,312.60 | 63,117.40 | 63,875.96 | 0.06 |
| 37 | n270-s20 | **31,195.00** | 31,361.20 | 31,281.80 | 31,583.46 | 0.28 |
| 38 | n275-s4 | 18,888.80 | 18,940.10 | **18,845.00** | 18,955.08 | -0.23 |
| 39 | n280-s100 | **31,886.20** | 32,197.80 | 33,334.00 | 33,904.13 | 4.54 |
| 40 | n284-s100 | **19,202.60** | 19,381.90 | 20,851.40 | 21,115.41 | 8.59 |
| 41 | n289-s20 | **71,639.00** | 71,748.80 | 72,553.60 | 73,040.41 | 1.28 |

Table 5: VNS results for the instances proposed in Toffolo et al (2018)

| # | Instance | T18 | | VNS | | Gap Best |
|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol | Best Sol. | Avg. Sol. | |
| 42 | n294-s4 | 47,632.80 | 50,338.40 | **43,882.80** | **44,194.92** | -7.87 |
| 43 | n298-s100 | **31,502.60** | 31,715.40 | 31,763.20 | 32,346.41 | 0.83 |
| 44 | n303-s4 | 22,711.40 | 22,832.10 | **22,010.80** | **22,164.29** | -3.08 |
| 45 | n308-s20 | **23,324.00** | 23,563.80 | 26,376.20 | 26,855.21 | 13.09 |
| 46 | n313-s4 | **73,087.00** | 75,694.00 | 73,592.40 | 74,131.13 | 0.69 |
| 47 | n317-s100 | **58,312.80** | 58,438.30 | 59,066.00 | 59,279.34 | 1.29 |
| 48 | n322-s20 | 30,819.20 | 31,153.40 | **29,840.80** | **30,007.93** | -3.17 |
| 49 | n327-s4 | **26,801.80** | 26,987.70 | 27,289.20 | 27,691.76 | 1.82 |
| 50 | n331-s100 | **28,260.20** | 28,513.70 | 29,871.60 | 31,720.21 | 5.70 |
| 51 | n336-s20 | **122,533.40** | 123,044.30 | 126,866.40 | 127,922.66 | 3.54 |
| 52 | n344-s4 | 41,666.20 | 45,303.00 | **39,384.20** | **39,638.38** | -5.48 |
| 53 | n351-s20 | 26,021.00 | 26,112.10 | **25,296.20** | **25,562.09** | -2.79 |
| 54 | n359-s100 | **42,286.60** | 42,537.90 | 43,255.00 | 44,229.89 | 2.29 |
| 55 | n367-s20 | **22,101.60** | 22,322.00 | 23,251.40 | 23,893.86 | 5.20 |
| 56 | n376-s100 | **111,476.20** | 111,893.70 | 112,244.00 | 112,682.29 | 0.69 |
| 57 | n384-s4 | **45,470.40** | 45,656.70 | 45,763.00 | 46,255.79 | 0.64 |
| 58 | n393-s100 | **36,519.40** | 36,832.40 | 36,578.40 | 37,070.42 | 0.16 |
| 59 | n401-s20 | **53,452.00** | 53,738.00 | 55,386.00 | 56,045.94 | 3.62 |
| 60 | n411-s4 | **18,803.60** | 19,278.40 | 19,598.40 | 20,090.39 | 4.23 |
| 61 | n420-s4 | 93,630.60 | 95,750.60 | **90,625.40** | **91,184.38** | -3.21 |
| 62 | n429-s100 | **56,636.00** | 56,865.50 | 56,933.40 | 57,496.13 | 0.53 |
| 63 | n439-s20 | **35,016.40** | 35,549.20 | 36,081.80 | 36,231.94 | 3.04 |
| 64 | n449-s100 | **41,570.80** | 41,953.50 | 45,144.60 | 46,335.62 | 8.60 |
| 65 | n459-s20 | 24,685.40 | 25,046.40 | **23,914.60** | **24,471.00** | -3.12 |
| 66 | n469-s4 | **189,868.80** | 190,409.40 | 192,272.80 | 192,972.46 | 1.27 |
| 67 | n480-s4 | 89,458.20 | 97,438.80 | **86,931.00** | **87,946.56** | -2.83 |
| 68 | n491-s20 | **45,883.40** | 46,023.20 | 47,050.80 | 47,740.12 | 2.54 |
| 69 | n502-s100 | **54,261.80** | 54,480.60 | 54,862.00 | 55,584.96 | 1.11 |
| 70 | n513-s100 | **24,472.40** | 24,686.70 | 25,411.00 | 25,988.33 | 3.84 |
| 71 | n524-s4 | 131,259.60 | 133,072.80 | **127,673.60** | **128,459.76** | -2.73 |
| 72 | n536-s20 | **71,191.00** | 71,373.20 | 72,526.00 | 72,998.75 | 1.88 |
| 73 | n548-s100 | **62,789.20** | 63,100.80 | 64,328.60 | 65,750.51 | 2.45 |
| 74 | n561-s4 | 51,781.60 | 52,142.10 | **43,379.40** | **44,107.02** | -16.23 |
| 75 | n573-s20 | **39,521.00** | 39,728.10 | 41,972.00 | 42,602.92 | 6.20 |
| 76 | n586-s20 | **144,678.00** | 145,081.50 | 147,110.20 | 148,211.75 | 1.68 |
| 77 | n599-s4 | **96,209.40** | 96,508.80 | 96,723.20 | 97,406.92 | 0.53 |
| 78 | n613-s100 | **53,891.00** | 54,242.50 | 56,210.80 | 56,956.66 | 4.30 |
| 79 | n627-s4 | **59,217.40** | 60,552.00 | 61,012.80 | 63,161.27 | 3.03 |
| 80 | n641-s20 | **56,630.00** | 56,985.90 | 63,565.00 | 64,926.79 | 12.25 |
| 81 | n655-s100 | **73,186.60** | 73,269.50 | 73,606.60 | 74,323.36 | 0.57 |
| 82 | n670-s4 | **98,178.20** | 98,832.00 | 99,978.20 | 101,009.82 | 1.83 |
| 83 | n685-s20 | **63,970.60** | 64,405.40 | 64,674.80 | 65,294.12 | 1.10 |
| 84 | n701-s100 | **68,307.80** | 69,105.00 | 74,385.80 | 76,024.13 | 8.90 |
| 85 | n716-s20 | **38,738.60** | 39,078.70 | 39,836.40 | 40,508.26 | 2.83 |
| 86 | n733-s4 | 141,618.40 | 155,822.30 | **135,863.80** | **136,737.58** | -4.06 |
| 87 | n749-s100 | **55,881.00** | 56,116.00 | 58,838.40 | 59,347.20 | 5.29 |
| 88 | n766-s100 | **105,462.80** | 106,062.00 | 109,687.20 | 111,092.95 | 4.01 |
| 89 | n783-s4 | **61,231.80** | 61,655.60 | 66,011.40 | 68,294.08 | 7.81 |
| 90 | n801-s20 | **60,632.80** | 61,576.70 | 68,920.20 | 70,526.68 | 13.67 |
| 91 | n819-s100 | **124,521.60** | 125,217.10 | 126,452.40 | 127,224.18 | 1.55 |
| 92 | n837-s4 | **133,712.00** | 134,346.00 | 136,281.80 | 137,573.93 | 1.92 |
| 93 | n856-s20 | 87,743.80 | 88,029.30 | **86,927.40** | 88,432.09 | -0.93 |
| 94 | n876-s20 | **90,187.60** | 90,740.00 | 96,814.40 | 98,233.59 | 7.35 |
| 95 | n895-s4 | **41,907.60** | 42,107.50 | 47,060.80 | 49,482.21 | 12.30 |
| 96 | n916-s100 | **240,702.40** | 241,110.20 | 243,804.40 | 245,225.90 | 1.29 |
| 97 | n936-s4 | 154,993.20 | 158,350.00 | **126,607.80** | **127,176.93** | -18.31 |
| 98 | n957-s100 | **73,133.60** | 73,653.70 | 76,759.40 | 78,119.99 | 4.96 |
| 99 | n979-s20 | **107,932.20** | 109,252.50 | 116,171.60 | 119,108.36 | 7.63 |
| 100 | n1001-s4 | **76,172.80** | 76,996.90 | 78,726.60 | 80,714.98 | 3.35 |
| | **Avg. Gap** | | | | | 1.23 |

The results of the combination of VNS and SP are reported in Table 6. In more detail, the first column gives the number of the instance and the second presents the name of the instance. Our results are compared to the results of Toffolo et al (2018), where the best values (out of ten runs) and the average values are provided in the third and fourth column, respectively. Three different variants of our proposed algorithm are tested in order to investigate how the VNS affects the solution quality of the SP. In particular, our first variant *Prop. algorithm 25%* executes the VNS for 25% of the overall computational time and allocates 75% to the SP. Alternatively, *Prop. algorithm 50%* equally divides the running time, while *Prop. algorithm 75%* assigns 450 seconds to the VNS and 150 seconds to the SP. Finally, in column *Gap Best* the percentage deviation of the best value found by our proposed algorithm with the value provided by the best approach is reported. The *Gap Best* is calculated as:

$$\frac{c_p - c^*}{c^*} \cdot 100, \tag{5}$$

where $c_p$ denotes the cost of the solution returned by the algorithm and $c^*$ expresses the cost of the best known solution. A negative gap indicates that the algorithm was able to improve the best solution so far. The best solution for each instance is accentuated in bold font.

From the results reported in Table 6, it follows that the proposed algorithm is competitive with respect to the existing approach for the SB-VRP. For 60 instances, the VNS-SP yields better results than just the VNS (see Table 5). However, in six of those cases this is due to the better performance of VNS, which means that SP does not give an advantage. Our proposed algorithm managed to improve 28 of the best solutions found by Toffolo et al (2018). In 21 cases, the VNS-SP was capable of further improving the best results of the VNS. The average gap was 1.25%, which is similar to the results of the VNS. The largest improvement was found for instance *n936-s4* with 17.68%, which is closely followed by an improvement of 16.05% for data set *n548-s100*.

Table 6: Our best results for the SB-VRP instances proposed in Toffolo et al (2018)

| # | Instance | T18 | | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Best | Avg. Sol | VNS | SP | VNS | SP | VNS | SP | Best |
| 1 | n101-s100 | 22,494.80 | 22,540.70 | 22,628.80 | **22,492.80** | 22,531.80 | **22,492.80** | 22,535.80 | **22,492.80** | -0.01 |
| 2 | n106-s20 | **23,615.40** | 23,661.40 | 23,745.60 | 23,669.60 | 23,688.80 | 23,656.60 | 23,702.60 | 23,647.60 | 0.14 |
| 3 | n110-s4 | 15,891.60 | 15,920.80 | **14,911.00** | **14,910.80** | **14,797.00** | **14,782.00** | **14,910.80** | **14,907.60** | -6.98 |
| 4 | n115-s100 | **12,191.00** | 12,303.00 | 12,596.60 | 12,566.00 | 12,606.60 | 12,538.40 | 12,581.60 | 12,558.80 | 2.85 |
| 5 | n120-s4 | 13,484.20 | 13,503.40 | 13,500.00 | 13,500.00 | 13,506.00 | 13,506.00 | **13,464.00** | **13,411.00** | -0.54 |
| 6 | n125-s20 | **36,461.60** | 36,515.20 | 36,726.40 | 36,483.60 | 36,555.80 | 36,540.00 | 36,628.40 | 36,592.20 | 0.06 |
| 7 | n129-s4 | 30,386.60 | 30,469.40 | **28,574.60** | **28,406.60** | **28,568.60** | **28,455.00** | **28,543.60** | **28,401.40** | -6.53 |
| 8 | n134-s100 | **9,896.60** | 9,960.20 | 10,216.20 | 10,113.20 | 10,204.20 | 10,141.80 | 10,127.60 | 10,068.60 | 1.74 |
| 9 | n139-s20 | **12,647.60** | 12,679.30 | 12,716.80 | 12,716.80 | 12,753.20 | 12,753.20 | 12,709.40 | 12,676.80 | 0.23 |
| 10 | n143-s100 | **14,736.40** | 14,824.00 | 16,010.20 | 15,923.00 | 15,289.20 | 15,269.20 | 15,843.00 | 15,841.00 | 3.62 |
| 11 | n148-s4 | 44,096.80 | 45,449.00 | **42,289.20** | **42,117.60** | **42,194.80** | **42,109.20** | **42,171.80** | **42,088.20** | -4.55 |
| 12 | n153-s20 | **14,770.80** | 14,816.00 | 14,833.20 | 14,820.80 | 14,819.60 | 14,819.20 | 14,819.20 | 14,819.20 | 0.33 |
| 13 | n157-s20 | **15,426.80** | 15,477.30 | 15,868.20 | 15,867.60 | 15,788.20 | 15,784.80 | 15,796.00 | 15,793.60 | 2.32 |
| 14 | n162-s100 | **13,719.40** | 13,808.00 | 14,038.00 | 14,027.60 | 14,017.60 | 14,006.80 | 13,963.80 | 13,927.00 | 1.51 |
| 15 | n167-s4 | 19,767.20 | 19,815.60 | **19,640.00** | **19,580.00** | **19,504.00** | **19,419.00** | **19,602.60** | **19,489.60** | -1.76 |
| 16 | n172-s100 | **30,024.00** | 30,032.50 | 30,127.20 | 30,027.60 | 30,081.80 | 30,046.00 | 30,102.80 | 30,078.00 | 0.01 |
| 17 | n176-s4 | 46,707.60 | 49,130.30 | **45,204.60** | **45,129.00** | **45,432.80** | **45,235.80** | **45,417.20** | **45,068.80** | -3.51 |
| 18 | n181-s20 | **22,091.00** | 22,143.20 | 22,522.40 | 22,416.20 | 22,435.40 | 22,346.80 | 22,613.00 | 22,403.40 | 1.16 |
| 19 | n186-s4 | 25,553.40 | 25,651.50 | **23,064.00** | **22,994.00** | **22,890.00** | **22,837.00** | **22,850.00** | **22,713.00** | -11.12 |
| 20 | n190-s100 | **15,359.40** | 15,463.50 | 17,104.40 | 17,104.40 | 17,104.40 | 17,104.40 | 17,104.40 | 17,104.40 | 11.36 |
| 21 | n195-s20 | 37,695.80 | 37,900.70 | 37,818.00 | **37,526.60** | 37,802.60 | **37,482.20** | **37,621.00** | **37,280.80** | -1.10 |
| 22 | n200-s100 | **50,592.40** | 50,723.50 | 51,473.00 | 51,127.20 | 51,388.00 | 51,322.60 | 51,242.00 | 51,168.00 | 1.06 |
| 23 | n204-s4 | 20,068.60 | 20,133.10 | **19,231.00** | **19,203.40** | **19,174.60** | **19,074.60** | **19,119.80** | **18,965.00** | -5.50 |
| 24 | n209-s20 | **26,658.80** | 26,738.30 | 27,304.00 | 27,179.60 | 27,153.80 | 27,082.40 | 27,185.00 | 27,087.80 | 1.59 |
| 25 | n214-s100 | **10,758.00** | 10,822.30 | 11,305.00 | 11,248.00 | 11,093.00 | 11,074.00 | 11,206.40 | 11,178.80 | 2.94 |
| 26 | n219-s4 | 100,729.00 | 101,295.40 | **99,330.60** | **98,925.80** | **99,128.00** | **98,938.60** | **99,254.20** | **98,913.20** | -1.80 |
| 27 | n223-s20 | **36,459.00** | 36,704.40 | 37,590.80 | 37,170.80 | 37,424.40 | 36,997.40 | 36,781.40 | 36,606.60 | 0.40 |
| 28 | n228-s20 | 22,431.20 | 22,649.60 | 22,611.60 | **22,399.00** | 22,436.00 | **22,215.20** | **22,268.60** | **22,227.80** | -0.96 |
| 29 | n233-s4 | 18,954.60 | 19,258.70 | 19,009.80 | **18,916.20** | **18,799.80** | **18,700.00** | **18,711.40** | **18,672.80** | -1.49 |
| 30 | n237-s100 | **25,234.20** | 25,454.60 | 27,589.60 | 27,433.20 | 27,015.40 | 27,004.20 | 27,006.20 | 27,000.20 | 7.00 |
| 31 | n242-s20 | **62,171.20** | 62,482.20 | 63,788.60 | 63,076.60 | 63,209.60 | 62,695.80 | 62,836.00 | 62,436.40 | 0.43 |
| 32 | n247-s100 | **28,058.60** | 28,221.80 | 29,159.20 | 28,719.60 | 29,061.00 | 28,549.60 | 28,827.20 | 28,827.20 | 1.75 |
| 33 | n251-s4 | **37,218.60** | 41,014.90 | 37,745.40 | 37,648.60 | 37,530.80 | 37,530.80 | 37,696.20 | 37,479.60 | 0.70 |
| 34 | n256-s4 | 21,687.80 | 22,023.10 | **19,296.00** | **19,137.00** | **19,092.00** | **19,012.00** | **19,147.00** | **19,076.00** | -12.34 |
| 35 | n261-s100 | **20,986.60** | 21,136.90 | 22,716.00 | 22,716.00 | 22,413.00 | 22,413.00 | 22,391.80 | 22,391.80 | 6.70 |
| 36 | n266-s20 | **63,079.20** | 63,312.60 | 64,320.20 | 63,700.20 | 63,862.60 | 63,837.60 | 63,723.00 | 63,379.00 | 0.48 |
| 37 | n270-s20 | **31,195.00** | 31,361.20 | 31,792.60 | 31,600.00 | 31,612.00 | 31,273.40 | 31,514.00 | 31,357.80 | 0.25 |
| 38 | n275-s4 | 18,888.80 | 18,940.10 | 18,954.20 | 18,893.00 | 18,936.20 | **18,848.20** | **18,867.40** | **18,821.20** | -0.36 |
| 39 | n280-s100 | **31,886.20** | 32,197.80 | 33,558.20 | 33,504.80 | 33,608.40 | 33,326.60 | 33,561.60 | 33,300.00 | 4.43 |
| 40 | n284-s100 | **19,202.60** | 19,381.90 | 21,132.00 | 21,084.80 | 21,126.40 | 21,126.40 | 20,766.80 | 20,766.80 | 8.15 |
| 41 | n289-s20 | **71,639.00** | 71,748.80 | 73,206.00 | 73,206.00 | 72,853.80 | 72,403.60 | 72,748.60 | 72,748.60 | 1.07 |
| 42 | n294-s4 | 47,632.80 | 50,338.40 | **43,743.80** | **43,537.40** | **43,582.20** | **43,425.40** | **43,618.00** | **43,440.40** | -8.83 |
| 43 | n298-s100 | **31,502.60** | 31,715.40 | 32,136.20 | 31,917.20 | 32,065.80 | 31,596.20 | 31,975.20 | 31,623.20 | 0.30 |
| 44 | n303-s4 | 22,711.40 | 22,832.10 | **22,129.60** | **22,039.60** | **22,049.00** | **21,984.80** | **21,958.80** | **21,958.80** | -3.31 |
| 45 | n308-s20 | **23,324.00** | 23,563.80 | 26,307.00 | 26,307.00 | 25,929.40 | 25,899.80 | 26,058.60 | 26,058.60 | 11.04 |
| 46 | n313-s4 | **73,087.00** | 75,694.00 | 74,273.40 | 73,557.60 | 73,707.40 | 73,631.80 | 73,663.20 | 73,561.40 | 0.64 |
| 47 | n317-s100 | **58,312.80** | 58,438.30 | 59,210.20 | 59,122.60 | 58,982.00 | 58,982.00 | 59,039.40 | 59,039.40 | 1.15 |
| 48 | n322-s20 | 30,819.20 | 31,153.40 | **30,052.60** | **29,984.60** | **29,771.80** | **29,759.80** | **29,899.20** | **29,871.60** | -3.44 |
| 49 | n327-s4 | **26,801.80** | 26,987.70 | 27,065.60 | 27,032.60 | 27,047.60 | 27,000.60 | 27,006.60 | 26,912.60 | 0.41 |
| 50 | n331-s100 | **28,260.20** | 28,513.70 | 31,363.80 | 31,363.80 | 30,363.00 | 30,288.00 | 30,824.20 | 30,524.40 | 7.18 |
| 51 | n336-s20 | **122,533.40** | 123,044.30 | 127,912.20 | 126,921.20 | 127,505.20 | 125,370.00 | 126,891.20 | 126,107.60 | 2.31 |
| 52 | n344-s4 | 41,666.20 | 45,303.00 | **39,590.40** | **39,346.60** | **39,600.20** | **39,289.40** | **39,541.80** | **39,392.20** | -5.70 |
| 53 | n351-s20 | 26,021.00 | 26,112.10 | **25,563.00** | **25,469.60** | **25,351.40** | **25,309.40** | **25,447.20** | **25,447.20** | -2.73 |
| 54 | n359-s100 | **42,286.60** | 42,537.90 | 44,028.60 | 44,005.60 | 43,704.20 | 43,606.40 | 43,423.60 | 43,423.60 | 2.69 |
| 55 | n367-s20 | **22,101.60** | 22,322.00 | 24,431.20 | 24,431.20 | 23,797.60 | 23,785.60 | 23,779.80 | 23,779.80 | 7.59 |
| 56 | n376-s100 | **111,476.20** | 111,893.70 | 112,848.00 | 112,074.80 | 112,651.80 | 111,816.80 | 112,450.20 | 112,083.60 | 0.31 |
| 57 | n384-s4 | **45,470.40** | 45,656.70 | 45,972.80 | 45,792.00 | 45,906.20 | 45,906.20 | 45,821.00 | 45,821.00 | 0.71 |
| 58 | n393-s100 | **36,519.40** | 36,832.40 | 37,025.60 | 36,728.60 | 36,642.20 | 36,575.40 | 36,798.80 | 36,654.40 | 0.15 |
| 59 | n401-s20 | **53,452.00** | 53,738.00 | 55,774.20 | 55,567.00 | 55,131.00 | 54,793.40 | 54,865.00 | 54,678.00 | 2.29 |
| 60 | n411-s4 | **18,803.60** | 19,278.40 | 19,998.20 | 19,974.40 | 19,408.80 | 19,402.60 | 19,352.40 | 19,315.60 | 2.72 |
| 61 | n420-s4 | 93,630.60 | 95,750.60 | **90,993.60** | **90,235.60** | **91,006.60** | **90,008.20** | **90,648.40** | **90,104.20** | -3.87 |
| 62 | n429-s100 | **56,636.00** | 56,865.50 | 58,018.80 | 57,126.80 | 57,556.40 | 56,756.40 | 57,456.60 | 57,218.40 | 0.21 |
| 63 | n439-s20 | **35,016.40** | 35,549.20 | 36,622.00 | 36,375.20 | 36,314.20 | 36,145.00 | 36,117.00 | 36,032.40 | 2.90 |
| 64 | n449-s100 | **41,570.80** | 41,953.50 | 46,348.00 | 46,348.00 | 45,666.40 | 45,553.20 | 45,912.00 | 45,912.00 | 9.58 |
| 65 | n459-s20 | 24,685.40 | 25,046.40 | **24,585.00** | **24,563.00** | **24,442.40** | **24,279.60** | **23,931.40** | **23,840.40** | -3.42 |

Table 6: Our best results for the SB-VRP instances proposed in Toffolo et al (2018)

| # | Instance | T18 | | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Best | Avg. Sol | VNS | SP | VNS | SP | VNS | SP | Best |
| 66 | n469-s4 | **189,868.80** | 190,409.40 | 193,743.20 | 192,798.20 | 193,088.40 | 192,018.40 | 192,199.80 | 191,263.20 | 0.73 |
| 67 | n480-s4 | 89,458.20 | 97,438.80 | **87,475.80** | **87,475.80** | **87,204.20** | **87,204.20** | **87,059.00** | **87,059.00** | -2.68 |
| 68 | n491-s20 | **45,883.40** | 46,023.20 | 47,135.40 | 47,135.40 | 46,843.20 | 46,843.20 | 47,060.00 | 47,060.00 | 2.09 |
| 69 | n502-s100 | **54,261.80** | 54,480.60 | 55,732.80 | 55,666.80 | 55,328.60 | 54,952.40 | 55,126.40 | 55,025.40 | 1.27 |
| 70 | n513-s100 | **24,472.40** | 24,686.70 | 25,737.40 | 25,715.40 | 25,420.40 | 25,388.40 | 25,345.60 | 25,309.60 | 3.42 |
| 71 | n524-s4 | 131,259.60 | 133,072.80 | **129,234.20** | **128,744.20** | **128,354.80** | **126,967.00** | **127,768.20** | **127,449.40** | -3.27 |
| 72 | n536-s20 | **71,191.00** | 71,373.20 | 74,092.20 | 74,092.20 | 73,171.60 | 73,171.60 | 72,582.20 | 72,582.20 | 1.95 |
| 73 | n548-s100 | **62,789.20** | 63,100.80 | 66,245.40 | 66,210.00 | 65,457.00 | 65,439.00 | 64,872.00 | 64,698.80 | 3.04 |
| 74 | n561-s4 | 51,781.60 | 52,142.10 | **44,218.00** | **44,048.40** | **43,855.80** | **43,855.80** | **43,471.80** | **43,471.80** | -16.05 |
| 75 | n573-s20 | **39,521.00** | 39,728.10 | 43,355.00 | 43,355.00 | 42,588.40 | 42,588.40 | 42,185.20 | 42,185.20 | 6.74 |
| 76 | n586-s20 | **144,678.00** | 145,081.50 | 149,090.60 | 149,090.60 | 147,945.00 | 147,945.00 | 147,126.20 | 147,126.20 | 1.69 |
| 77 | n599-s4 | **96,209.40** | 96,508.80 | 98,451.60 | 98,451.60 | 97,411.60 | 97,411.60 | 97,346.00 | 97,346.00 | 1.18 |
| 78 | n613-s100 | **53,891.00** | 54,242.50 | 56,742.80 | 56,742.80 | 55,896.20 | 55,180.80 | 55,722.40 | 55,722.40 | 2.39 |
| 79 | n627-s4 | **59,217.40** | 60,552.00 | 63,461.40 | 63,452.40 | 63,083.20 | 63,013.60 | 61,951.00 | 61,951.00 | 4.62 |
| 80 | n641-s20 | **56,630.00** | 56,985.90 | 63,732.20 | 63,732.20 | 62,604.60 | 62,604.60 | 62,872.00 | 62,872.00 | 10.55 |
| 81 | n655-s100 | **73,186.60** | 73,269.50 | 75,334.20 | 75,334.20 | 74,609.00 | 74,609.00 | 74,308.60 | 74,308.60 | 1.53 |
| 82 | n670-s4 | **98,178.20** | 98,832.00 | 103,027.80 | 103,027.80 | 101,030.80 | 101,030.80 | 100,719.60 | 100,719.60 | 2.59 |
| 83 | n685-s20 | **63,970.60** | 64,405.40 | 65,811.80 | 65,811.80 | 65,500.20 | 65,414.00 | 65,198.80 | 65,198.80 | 1.92 |
| 84 | n701-s100 | **68,307.80** | 69,105.00 | 78,356.80 | 78,270.80 | 76,156.00 | 76,152.00 | 75,217.40 | 75,217.40 | 10.12 |
| 85 | n716-s20 | **38,738.60** | 39,078.70 | 41,503.00 | 41,503.00 | 40,486.00 | 40,473.00 | 40,269.40 | 40,269.40 | 3.95 |
| 86 | n733-s4 | 141,618.40 | 155,822.30 | **138,225.40** | **137,887.60** | **137,131.60** | **136,683.20** | **136,316.60** | **136,316.60** | -3.74 |
| 87 | n749-s100 | **55,881.00** | 56,116.00 | 60,193.60 | 60,193.60 | 58,991.40 | 58,991.40 | 58,916.20 | 58,916.20 | 5.43 |
| 88 | n766-s100 | **105,462.80** | 106,062.00 | 112,025.60 | 112,025.60 | 110,555.40 | 110,555.40 | 109,080.00 | 109,080.00 | 3.43 |
| 89 | n783-s4 | **61,231.80** | 61,655.60 | 68,442.80 | 68,439.80 | 67,379.00 | 67,379.00 | 66,518.60 | 66,518.60 | 8.63 |
| 90 | n801-s20 | **60,632.80** | 61,576.70 | 73,629.00 | 73,629.00 | 71,651.00 | 71,651.00 | 71,157.20 | 71,157.20 | 17.36 |
| 91 | n819-s100 | **124,521.60** | 125,217.10 | 129,761.40 | 129,761.40 | 128,144.80 | 128,144.80 | 127,319.40 | 127,319.40 | 2.25 |
| 92 | n837-s4 | **133,712.00** | 134,346.00 | 140,146.20 | 140,146.20 | 137,680.00 | 137,680.00 | 137,029.00 | 137,029.00 | 2.48 |
| 93 | n856-s20 | 87,743.80 | 88,029.30 | 91,707.00 | 91,703.00 | 88,013.40 | 88,013.40 | **87,570.20** | **87,570.20** | -0.20 |
| 94 | n876-s20 | **90,187.60** | 90,740.00 | 99,968.40 | 99,968.40 | 97,876.80 | 97,868.80 | 97,051.00 | 97,051.00 | 7.61 |
| 95 | n895-s4 | **41,907.60** | 42,107.50 | 51,954.20 | 51,954.20 | 48,846.60 | 48,846.60 | 48,839.20 | 48,839.20 | 16.54 |
| 96 | n916-s100 | **240,702.40** | 241,110.20 | 248,909.80 | 248,909.80 | 245,879.20 | 245,879.20 | 243,905.40 | 243,905.40 | 1.33 |
| 97 | n936-s4 | 154,993.20 | 158,350.00 | **131,680.00** | **131,680.00** | **128,290.40** | **128,290.40** | **127,595.60** | **127,595.60** | -17.68 |
| 98 | n957-s100 | **73,133.60** | 73,653.70 | 80,461.20 | 80,351.20 | 77,907.00 | 77,520.40 | 76,821.20 | 76,810.60 | 5.03 |
| 99 | n979-s20 | **107,932.20** | 109,252.50 | 121,944.60 | 121,944.60 | 118,955.80 | 118,955.80 | 117,943.40 | 117,943.40 | 9.28 |
| 100 | n1001-s4 | **76,172.80** | 76,996.90 | 82,513.60 | 82,513.60 | 80,079.00 | 80,079.00 | 79,739.40 | 79,739.40 | 4.68 |
| | **Avg. Gap** | | | | | | | | | 1.25 |

From the results presented in Table 6, we may infer that *Prop. algorithm 75%* is a better option for solving the SB-VRP than *Prop. algorithm 25%* as well as *Prop. algorithm 50%*. Although *Prop. algorithm 50%* achieved slightly better results for the instance *n110-s4* on the other instances the *Prop. algorithm 75%* provided better solutions. Based on the numerical results it seems that our approach performs well when the number of swap locations $\#V_s$ as well as the number of train customers $\#V_c^{Train}$ is relatively low (see e. g. *n936-s4*).

Table 7 presents a comparison in terms of average solutions between the results found by VNS-SP and those determined in Toffolo et al (2018). Our average results outperform the best results for 24 instances. The average gap is also relatively low with 2.33%.

Table 7: Our average results for the SB-VRP instances in Toffolo et al (2018)

| # | Instance | T18 | | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | VNS | SP | VNS | SP | VNS | SP | Best |
| 1 | n101-s100 | **22,494.80** | 22,540.70 | 22,703.16 | 22,523.94 | 22,624.12 | 22,525.70 | 22,668.50 | 22,515.86 | 0.09 |
| 2 | n106-s20 | **23,615.40** | 23,661.40 | 23,810.62 | 23,744.16 | 23,780.56 | 23,714.68 | 23,765.10 | 23,710.08 | 0.40 |
| 3 | n110-s4 | 15,891.60 | 15,920.80 | **14,958.06** | **14,943.08** | **14,983.96** | **14,945.66** | **14,994.38** | **14,946.56** | -5.97 |
| 4 | n115-s100 | **12,191.00** | 12,303.00 | 12,682.44 | 12,613.66 | 12,655.86 | 12,584.34 | 12,630.86 | 12,573.30 | 3.14 |
| 5 | n120-s4 | **13,484.20** | 13,503.40 | 13,582.10 | 13,560.00 | 13,546.50 | 13,530.00 | 13,537.30 | 13,506.40 | 0.16 |
| 6 | n125-s20 | **36,461.60** | 36,515.20 | 36,931.20 | 36,695.98 | 36,899.20 | 36,690.26 | 36,893.34 | 36,712.40 | 0.63 |
| 7 | n129-s4 | 30,386.60 | 30,469.40 | **28,735.94** | **28,494.98** | **28,684.12** | **28,527.78** | **28,714.98** | **28,519.24** | -6.23 |
| 8 | n134-s100 | **9,896.60** | 9,960.20 | 10,296.42 | 10,233.88 | 10,268.28 | 10,197.90 | 10,208.66 | 10,163.06 | 2.69 |
| 9 | n139-s20 | **12,647.60** | 12,679.30 | 12,936.60 | 12,883.66 | 12,916.42 | 12,897.60 | 12,840.72 | 12,790.00 | 1.13 |
| 10 | n143-s100 | **14,736.40** | 14,824.00 | 16,283.02 | 16,246.94 | 16,059.12 | 16,028.12 | 16,027.84 | 16,012.64 | 8.66 |
| 11 | n148-s4 | 44,096.80 | 45,449.00 | **42,363.58** | **42,183.78** | **42,309.68** | **42,158.64** | **42,277.38** | **42,142.72** | -4.43 |
| 12 | n153-s20 | **14,770.80** | 14,816.00 | 14,895.80 | 14,871.52 | 14,859.92 | 14,857.60 | 14,876.18 | 14,876.18 | 0.59 |
| 13 | n157-s20 | **15,426.80** | 15,477.30 | 15,944.14 | 15,926.42 | 15,897.86 | 15,891.84 | 15,881.88 | 15,876.66 | 2.92 |
| 14 | n162-s100 | **13,719.40** | 13,808.00 | 14,082.78 | 14,054.98 | 14,080.88 | 14,055.08 | 14,053.20 | 14,020.36 | 2.19 |
| 15 | n167-s4 | 19,767.20 | 19,815.60 | 20,039.56 | 20,021.46 | **19,713.08** | **19,658.72** | 19,831.36 | 19,772.66 | -0.55 |
| 16 | n172-s100 | **30,024.00** | 30,032.50 | 30,255.82 | 30,098.80 | 30,235.22 | 30,133.32 | 30,223.96 | 30,160.10 | 0.25 |
| 17 | n176-s4 | 46,707.60 | 49,130.30 | **45,876.68** | **45,526.92** | **45,945.38** | **45,573.72** | **45,617.68** | **45,394.56** | -2.81 |
| 18 | n181-s20 | **22,091.00** | 22,143.20 | 22,860.08 | 22,611.64 | 22,689.58 | 22,505.96 | 22,782.48 | 22,597.32 | 1.88 |
| 19 | n186-s4 | 25,553.40 | 25,651.50 | **23,336.46** | **23,183.06** | **23,108.20** | **22,956.70** | **23,067.86** | **22,924.06** | -10.29 |
| 20 | n190-s100 | **15,359.40** | 15,463.50 | 17,110.40 | 17,110.40 | 17,104.40 | 17,104.40 | 17,104.40 | 17,104.40 | 11.36 |
| 21 | n195-s20 | 37,695.80 | 37,900.70 | 38,192.26 | 37,764.72 | 38,040.40 | **37,587.06** | 37,923.48 | **37,546.36** | -0.40 |
| 22 | n200-s100 | **50,592.40** | 50,723.50 | 51,569.04 | 51,319.94 | 51,516.40 | 51,504.32 | 51,409.88 | 51,370.46 | 1.44 |
| 23 | n204-s4 | 20,068.60 | 20,133.10 | **19,424.48** | **19,325.08** | **19,384.84** | **19,279.30** | **19,313.94** | **19,216.80** | -4.24 |

Table 7: Our average results for the SB-VRP instances in Toffolo et al (2018)

| # | Instance | T18 | | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | VNS | SP | VNS | SP | VNS | SP | Best |
| 24 | n209-s20 | **26,658.80** | 26,738.30 | 27,687.32 | 27,594.22 | 27,494.12 | 27,410.10 | 27,517.34 | 27,489.28 | 2.82 |
| 25 | n214-s100 | **10,758.00** | 10,822.30 | 11,525.74 | 11,467.32 | 11,322.76 | 11,266.58 | 11,454.58 | 11,443.84 | 4.73 |
| 26 | n219-s4 | 100,729.00 | 101,295.40 | **99,445.46** | **99,019.20** | **99,384.08** | **99,002.22** | **99,351.50** | **98,983.72** | -1.73 |
| 27 | n223-s20 | **36,459.00** | 36,704.40 | 38,240.84 | 37,736.62 | 37,789.38 | 37,403.02 | 37,538.22 | 37,222.30 | 2.09 |
| 28 | n228-s20 | **22,431.20** | 22,649.60 | 22,792.26 | 22,577.70 | 22,621.60 | 22,468.80 | 22,639.32 | 22,486.36 | 0.17 |
| 29 | n233-s4 | 18,954.60 | 19,258.70 | 19,237.32 | 19,119.22 | 18,999.80 | **18,934.44** | 19,023.06 | 18,988.12 | -0.11 |
| 30 | n237-s100 | **25,234.20** | 25,454.60 | 27,979.52 | 27,886.00 | 27,768.60 | 27,713.94 | 27,606.16 | 27,567.96 | 9.25 |
| 31 | n242-s20 | **62,171.20** | 62,482.20 | 64,089.82 | 63,947.50 | 63,828.56 | 63,688.32 | 63,462.16 | 63,396.02 | 1.97 |
| 32 | n247-s100 | **28,058.60** | 28,221.80 | 29,418.16 | 29,090.54 | 29,257.66 | 29,129.78 | 29,218.84 | 29,146.28 | 3.68 |
| 33 | n251-s4 | **37,218.60** | 41,014.90 | 37,922.80 | 37,808.08 | 37,844.82 | 37,807.34 | 37,830.40 | 37,772.58 | 1.49 |
| 34 | n256-s4 | 21,687.80 | 22,023.10 | **19,561.90** | **19,300.60** | **19,490.90** | **19,265.60** | **19,338.10** | **19,183.20** | -11.55 |
| 35 | n261-s100 | **20,986.60** | 21,136.90 | 22,928.86 | 22,928.86 | 22,763.20 | 22,751.10 | 22,715.54 | 22,705.64 | 8.19 |
| 36 | n266-s20 | **63,079.20** | 63,312.60 | 64,763.32 | 64,189.52 | 64,484.44 | 64,034.38 | 64,334.74 | 63,693.40 | 0.97 |
| 37 | n270-s20 | **31,195.00** | 31,361.20 | 32,058.08 | 31,825.06 | 31,845.54 | 31,584.06 | 31,837.72 | 31,719.58 | 1.25 |
| 38 | n275-s4 | 18,888.80 | 18,940.10 | 19,034.90 | 18,971.26 | 19,002.86 | 18,937.16 | 18,928.52 | **18,883.46** | -0.03 |
| 39 | n280-s100 | **31,886.20** | 32,197.80 | 34,118.52 | 34,036.68 | 33,963.72 | 33,854.28 | 33,900.10 | 33,845.70 | 6.15 |
| 40 | n284-s100 | **19,202.60** | 19,381.90 | 21,493.88 | 21,413.60 | 21,310.80 | 21,276.90 | 21,168.70 | 21,158.82 | 10.19 |
| 41 | n289-s20 | **71,639.00** | 71,748.80 | 73,659.00 | 73,617.46 | 73,362.12 | 73,302.20 | 73,215.56 | 73,215.56 | 2.20 |
| 42 | n294-s4 | 47,632.80 | 50,338.40 | **44,231.94** | **43,785.94** | **44,114.50** | **43,826.92** | **43,989.24** | **43,707.26** | -8.24 |
| 43 | n298-s100 | **31,502.60** | 31,715.40 | 32,712.78 | 32,274.54 | 32,564.16 | 32,275.40 | 32,196.78 | 31,965.14 | 1.47 |
| 44 | n303-s4 | 22,711.40 | 22,832.10 | **22,205.46** | **22,132.36** | **22,155.36** | **22,121.10** | **22,072.44** | **22,072.44** | -2.81 |
| 45 | n308-s20 | **23,324.00** | 23,563.80 | 26,692.44 | 26,686.68 | 26,557.62 | 26,547.74 | 26,367.10 | 26,364.00 | 13.03 |
| 46 | n313-s4 | **73,087.00** | 75,694.00 | 74,647.38 | 74,187.16 | 74,304.40 | 74,144.96 | 74,245.52 | 74,125.16 | 1.42 |
| 47 | n317-s100 | **58,312.80** | 58,438.30 | 59,359.34 | 59,265.14 | 59,159.80 | 59,145.44 | 59,137.24 | 59,137.24 | 1.41 |
| 48 | n322-s20 | 30,819.20 | 31,153.40 | **30,150.30** | **30,050.66** | **30,017.50** | **29,957.72** | **30,007.42** | **29,986.58** | -2.80 |
| 49 | n327-s4 | **26,801.80** | 26,987.70 | 27,646.60 | 27,552.72 | 27,523.52 | 27,449.12 | 27,537.20 | 27,454.00 | 2.42 |
| 50 | n331-s100 | **28,260.20** | 28,513.70 | 31,867.90 | 31,866.60 | 31,158.38 | 31,127.68 | 31,243.90 | 31,140.22 | 10.15 |
| 51 | n336-s20 | **122,533.40** | 123,044.30 | 128,960.88 | 128,147.02 | 128,200.12 | 126,821.56 | 127,660.58 | 126,966.96 | 3.50 |
| 52 | n344-s4 | 41,666.20 | 45,303.00 | **39,778.90** | **39,659.04** | **39,746.74** | **39,631.38** | **39,670.62** | **39,655.18** | -4.88 |
| 53 | n351-s20 | 26,021.00 | 26,112.10 | **25,772.00** | **25,708.08** | **25,639.94** | **25,626.24** | **25,650.40** | **25,650.40** | -1.52 |
| 54 | n359-s100 | **42,286.60** | 42,537.90 | 44,401.44 | 44,361.88 | 44,114.50 | 44,070.10 | 43,872.28 | 43,870.54 | 3.75 |
| 55 | n367-s20 | **22,101.60** | 22,322.00 | 24,979.56 | 24,973.88 | 24,545.22 | 24,540.54 | 24,222.44 | 24,222.26 | 9.60 |
| 56 | n376-s100 | **111,476.20** | 111,893.70 | 113,314.78 | 112,341.84 | 113,108.98 | 112,379.18 | 112,712.00 | 112,363.56 | 0.78 |
| 57 | n384-s4 | **45,470.40** | 45,656.70 | 46,286.28 | 46,211.26 | 46,198.36 | 46,187.76 | 46,073.22 | 46,073.22 | 1.33 |
| 58 | n393-s100 | **36,519.40** | 36,832.40 | 37,349.22 | 37,091.02 | 37,035.32 | 36,836.22 | 36,964.48 | 36,898.68 | 0.87 |
| 59 | n401-s20 | **53,452.00** | 53,738.00 | 56,159.82 | 55,934.52 | 55,498.64 | 55,269.14 | 55,551.14 | 55,481.88 | 3.40 |
| 60 | n411-s4 | **18,803.60** | 19,278.40 | 20,250.94 | 20,198.62 | 19,898.94 | 19,839.78 | 19,755.32 | 19,676.14 | 4.64 |
| 61 | n420-s4 | 93,630.60 | 95,750.60 | **91,618.22** | **90,521.96** | **91,476.38** | **90,529.14** | **91,276.00** | **90,478.98** | -3.37 |
| 62 | n429-s100 | **56,636.00** | 56,865.50 | 58,394.74 | 57,763.38 | 58,020.28 | 57,664.96 | 57,675.62 | 57,576.80 | 1.66 |
| 63 | n439-s20 | **35,016.40** | 35,549.20 | 36,718.46 | 36,533.18 | 36,388.02 | 36,260.30 | 36,406.40 | 36,345.48 | 3.55 |
| 64 | n449-s100 | **41,570.80** | 41,953.50 | 47,047.90 | 47,044.58 | 46,445.06 | 46,418.90 | 46,521.08 | 46,521.08 | 11.66 |
| 65 | n459-s20 | 24,685.40 | 25,046.40 | 24,833.12 | 24,759.06 | 24,704.02 | **24,593.64** | **24,451.64** | **24,404.00** | -1.14 |
| 66 | n469-s4 | **189,868.80** | 190,409.40 | 195,111.84 | 193,982.08 | 193,796.54 | 192,973.12 | 193,532.86 | 192,814.30 | 1.55 |
| 67 | n480-s4 | 89,458.20 | 97,438.80 | **88,394.82** | **88,394.82** | **87,792.56** | **87,792.56** | **87,606.70** | **87,606.70** | -2.07 |
| 68 | n491-s20 | **45,883.40** | 46,023.20 | 47,575.32 | 47,575.32 | 47,214.82 | 47,214.82 | 47,331.78 | 47,331.78 | 2.90 |
| 69 | n502-s100 | **54,261.80** | 54,480.60 | 56,100.10 | 55,934.76 | 55,653.44 | 55,479.38 | 55,562.90 | 55,390.16 | 2.08 |
| 70 | n513-s100 | **24,472.40** | 24,686.70 | 26,055.48 | 25,983.10 | 25,908.80 | 25,798.42 | 25,756.70 | 25,738.50 | 5.17 |
| 71 | n524-s4 | 131,259.60 | 133,072.80 | **130,033.30** | **129,228.78** | **128,917.10** | **127,969.56** | **128,662.36** | **128,066.92** | -2.51 |
| 72 | n536-s20 | **71,191.00** | 71,373.20 | 74,510.12 | 74,510.12 | 73,618.50 | 73,618.50 | 73,112.92 | 73,112.92 | 2.70 |
| 73 | n548-s100 | **62,789.20** | 63,100.80 | 66,817.86 | 66,771.80 | 66,152.08 | 66,121.98 | 65,493.26 | 65,460.12 | 4.25 |
| 74 | n561-s4 | 51,781.60 | 52,142.10 | **44,693.12** | **44,599.04** | **44,364.50** | **44,364.50** | **43,899.66** | **43,899.66** | -15.22 |
| 75 | n573-s20 | **39,521.00** | 39,728.10 | 43,621.98 | 43,621.98 | 42,966.08 | 42,966.08 | 42,610.18 | 42,610.18 | 7.82 |
| 76 | n586-s20 | **144,678.00** | 145,081.50 | 149,755.66 | 149,755.66 | 148,565.70 | 148,565.70 | 148,285.06 | 148,285.06 | 2.49 |
| 77 | n599-s4 | **96,209.40** | 96,508.80 | 98,928.04 | 98,928.04 | 98,096.94 | 98,096.94 | 97,814.48 | 97,814.48 | 1.67 |
| 78 | n613-s100 | **53,891.00** | 54,242.50 | 57,672.20 | 57,669.74 | 56,543.46 | 56,352.82 | 56,548.24 | 56,522.30 | 4.57 |
| 79 | n627-s4 | **59,217.40** | 60,552.00 | 63,874.38 | 63,872.14 | 63,706.02 | 63,689.98 | 63,097.84 | 63,097.54 | 6.55 |
| 80 | n641-s20 | **56,630.00** | 56,985.90 | 64,449.36 | 64,388.86 | 63,384.48 | 63,361.16 | 63,461.42 | 63,456.48 | 11.89 |
| 81 | n655-s100 | **73,186.60** | 73,269.50 | 75,839.84 | 75,839.84 | 74,969.66 | 74,969.66 | 74,716.08 | 74,716.08 | 2.09 |
| 82 | n670-s4 | **98,178.20** | 98,832.00 | 104,375.30 | 104,375.30 | 102,439.26 | 102,439.26 | 102,233.68 | 102,229.36 | 4.13 |
| 83 | n685-s20 | **63,970.60** | 64,405.40 | 66,485.34 | 66,468.88 | 65,909.62 | 65,890.46 | 65,565.16 | 65,565.16 | 2.49 |
| 84 | n701-s100 | **68,307.80** | 69,105.00 | 79,077.60 | 79,069.00 | 77,371.18 | 77,351.60 | 76,239.88 | 76,239.88 | 11.61 |
| 85 | n716-s20 | **38,738.60** | 39,078.70 | 41,902.46 | 41,898.46 | 41,170.84 | 41,157.34 | 40,992.32 | 40,991.62 | 5.82 |
| 86 | n733-s4 | 141,618.40 | 155,822.30 | **138,903.00** | **138,416.74** | **137,740.08** | **137,370.62** | **136,954.70** | **136,836.82** | -3.38 |
| 87 | n749-s100 | **55,881.00** | 56,116.00 | 60,620.90 | 60,620.90 | 59,889.50 | 59,888.00 | 59,466.88 | 59,466.88 | 6.42 |
| 88 | n766-s100 | **105,462.80** | 106,062.00 | 112,831.60 | 112,796.56 | 111,336.34 | 111,336.34 | 110,491.82 | 110,491.82 | 4.77 |
| 89 | n783-s4 | **61,231.80** | 61,655.60 | 68,984.36 | 68,952.80 | 67,997.52 | 67,996.76 | 67,336.00 | 67,336.00 | 9.97 |
| 90 | n801-s20 | **60,632.80** | 61,576.70 | 73,768.22 | 73,768.22 | 72,021.86 | 72,004.64 | 71,553.28 | 71,552.36 | 18.01 |
| 91 | n819-s100 | **124,521.60** | 125,217.10 | 130,738.92 | 130,738.92 | 129,068.54 | 129,068.54 | 128,241.58 | 128,241.58 | 2.99 |
| 92 | n837-s4 | **133,712.00** | 134,346.00 | 140,928.12 | 140,928.12 | 138,679.14 | 138,679.14 | 138,161.38 | 138,161.38 | 3.33 |
| 93 | n856-s20 | **87,743.80** | 88,029.30 | 92,471.18 | 92,411.40 | 90,091.14 | 90,081.14 | 88,876.42 | 88,876.42 | 1.29 |
| 94 | n876-s20 | **90,187.60** | 90,740.00 | 100,624.34 | 100,624.24 | 99,148.62 | 99,146.22 | 98,419.12 | 98,419.12 | 9.13 |
| 95 | n895-s4 | **41,907.60** | 42,107.50 | 51,954.20 | 51,954.20 | 49,929.00 | 49,929.00 | 49,377.42 | 49,376.94 | 17.82 |
| 96 | n916-s100 | **240,702.40** | 241,110.20 | 251,508.78 | 251,492.30 | 246,570.10 | 246,570.10 | 245,703.86 | 245,703.86 | 2.08 |
| 97 | n936-s4 | 154,993.20 | 158,350.00 | **133,256.30** | **133,256.30** | **129,348.48** | **129,348.48** | **128,251.18** | **128,251.18** | -17.25 |
| 98 | n957-s100 | **73,133.60** | 73,653.70 | 81,708.78 | 81,592.90 | 78,977.64 | 78,937.58 | 78,251.88 | 78,218.68 | 6.95 |
| 99 | n979-s20 | **107,932.20** | 109,252.50 | 123,228.70 | 123,228.70 | 120,337.38 | 120,336.74 | 120,069.96 | 120,069.96 | 11.25 |
| 100 | n1001-s4 | **76,172.80** | 76,996.90 | 83,013.28 | 83,013.28 | 80,634.80 | 80,634.80 | 80,065.06 | 80,065.06 | 5.11 |
| | **Avg. Gap** | | | | | | | | | 2.33 |

In line with the experiments of Toffolo et al (2018), we also conducted experiments with an extended running time of 1,200 seconds. The results are presented in Table 8. The *Gap Best* is calculated with respect to the results after 600 seconds (sec.) and after 1,200 sec. With this longer running time the algorithm is capable to further improve the average gap to 0.88%. In addition, 31 best solutions can be outperformed, where in comparison of our best results in Table 6 three new best solutions can be found in *n393-s100*, *n266-s20* as well as *n125-s20*.

Table 8: Our best results (1,200 seconds) for the instances given in Toffolo et al (2018)

| # | Testinstance | T18 (600 sec.) | | T18 (1,200 sec.) | | Prop. algorithm 75% | | Gap | Gap |
|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Avg. Sol. | VNS | SP | Best 600 sec. | Best 1,200 sec. |
| 1 | n101-s100 | 22,494.80 | 22,540.70 | **22,492.80** | 22,506.68 | **22,492.80** | **22,492.80** | -0.01 | 0.00 |
| 2 | n106-s20 | **23,615.40** | 23,661.40 | 23,643.09 | 23,629.60 | 23,669.60 | 23,659.00 | 0.18 | 0.07 |
| 3 | n110-s4 | 15,891.60 | 15,920.80 | 15,891.60 | 15,897.86 | **14,869.00** | **14,840.00** | -6.62 | -6.62 |
| 4 | n115-s100 | 12,191.00 | 12,303.00 | **12,186.20** | 12,223.85 | 12,442.00 | 12,434.40 | 2.00 | 2.04 |
| 5 | n120-s4 | 13,484.20 | 13,503.40 | 13,390.60 | 13,484.20 | **13,380.00** | **13,380.00** | -0.77 | -0.08 |
| 6 | n125-s20 | 36,461.60 | 36,515.20 | 36,439.20 | 36,476.56 | **36,350.20** | **36,282.80** | -0.49 | -0.43 |
| 7 | n129-s4 | 30,386.60 | 30,469.40 | 30,355.40 | 30,408.99 | **28,507.40** | **28,342.40** | -6.73 | -6.63 |
| 8 | n134-s100 | 9,896.60 | 9,960.20 | **9,890.40** | 9,918.03 | 10,192.20 | 10,103.00 | 2.09 | 2.15 |
| 9 | n139-s20 | **12,647.60** | 12,679.30 | **12,647.60** | 12,660.45 | 12,699.20 | 12,690.80 | 0.34 | 0.34 |
| 10 | n143-s100 | **14,736.40** | 14,824.00 | **14,736.40** | 14,754.40 | 15,724.00 | 15,609.40 | 5.92 | 5.92 |
| 11 | n148-s4 | 44,096.80 | 45,449.00 | 43,252.00 | 44,030.44 | **42,151.80** | **42,079.20** | -4.58 | -2.71 |
| 12 | n153-s20 | **14,770.80** | 14,816.00 | 14,773.20 | 14,797.89 | 14,812.00 | 14,812.00 | 0.28 | 0.26 |
| 13 | n157-s20 | 15,426.80 | 15,477.30 | **15,376.00** | 15,417.12 | 15,790.40 | 15,787.20 | 2.34 | 2.67 |
| 14 | n162-s100 | **13,719.40** | 13,808.00 | **13,719.40** | 13,745.48 | 13,960.80 | 13,930.00 | 1.54 | 1.54 |
| 15 | n167-s4 | 19,767.20 | 19,815.60 | 19,695.20 | 19,777.78 | **19,262.60** | **19,253.60** | -2.60 | -2.24 |
| 16 | n172-s100 | **30,024.00** | 30,032.50 | **30,024.00** | 30,026.21 | 30,037.20 | 30,037.20 | 0.04 | 0.04 |
| 17 | n176-s4 | 46,707.60 | 49,130.30 | 45,484.00 | 46,866.35 | **45,339.60** | **44,880.80** | -3.91 | -1.33 |
| 18 | n181-s20 | 22,091.00 | 22,143.20 | **22,056.80** | 22,104.73 | 22,461.00 | 22,260.00 | 0.77 | 0.92 |
| 19 | n186-s4 | 25,553.40 | 25,651.50 | 25,131.80 | 25,491.21 | **22,890.00** | **22,672.00** | -11.28 | -9.79 |
| 20 | n190-s100 | 15,359.40 | 15,463.50 | **15,301.00** | 15,372.54 | 16,589.80 | 16,588.80 | 8.00 | 8.42 |
| 21 | n195-s20 | 37,695.80 | 37,900.70 | 37,471.20 | 37,658.32 | **37,587.80** | **37,325.80** | -0.98 | -0.39 |
| 22 | n200-s100 | 50,592.40 | 50,723.50 | **50,427.00** | 50,579.62 | 51,280.60 | 51,280.60 | 1.36 | 1.69 |
| 23 | n204-s4 | 20,068.60 | 20,133.10 | 20,043.40 | 20,079.93 | **18,976.80** | **18,917.60** | -5.74 | -5.62 |
| 24 | n209-s20 | **26,658.80** | 26,738.30 | 26,668.98 | 26,668.98 | 27,071.80 | 27,055.80 | 1.49 | 1.45 |
| 25 | n214-s100 | 10,758.00 | 10,822.30 | **10,704.80** | 10,763.21 | 11,210.80 | 11,091.40 | 3.10 | 3.61 |
| 26 | n219-s4 | 100,729.00 | 101,295.40 | 100,649.00 | 100,978.25 | **99,086.60** | **98,866.80** | -1.85 | -1.77 |
| 27 | n223-s20 | 36,459.00 | 36,704.40 | **36,417.00** | 36,516.64 | 36,977.00 | 36,640.60 | 0.50 | 0.61 |
| 28 | n228-s20 | 22,431.20 | 22,649.60 | 22,427.60 | 22,515.39 | 22,462.20 | **22,281.40** | -0.67 | -0.65 |
| 29 | n233-s4 | 18,954.60 | 19,258.70 | 18,941.80 | 19,107.78 | **18,702.80** | **18,667.40** | -1.52 | -1.45 |
| 30 | n237-s100 | 25,234.20 | 25,454.60 | **25,205.40** | 25,279.33 | 27,134.00 | 27,115.00 | 7.45 | 7.58 |
| 31 | n242-s20 | 62,171.20 | 62,482.20 | **62,108.60** | 62,266.32 | 62,975.60 | 62,975.60 | 1.29 | 1.40 |
| 32 | n247-s100 | 28,058.60 | 28,221.80 | **27,911.40** | 28,058.08 | 28,637.00 | 28,565.20 | 1.81 | 2.34 |
| 33 | n251-s4 | 37,218.60 | 41,014.90 | **37,010.80** | 37,626.60 | 37,427.80 | 37,332.40 | 0.31 | 0.87 |
| 34 | n256-s4 | 21,687.80 | 22,023.10 | 19,426.80 | 21,639.21 | **18,990.00** | **18,990.00** | -12.44 | -2.25 |
| 35 | n261-s100 | **20,986.60** | 21,136.90 | **20,986.80** | 21,041.61 | 22,283.00 | 22,283.00 | 6.18 | 6.18 |
| 36 | n266-s20 | 63,079.20 | 63,312.60 | 62,917.80 | 63,118.29 | 63,338.20 | **62,784.20** | -0.47 | -0.21 |
| 37 | n270-s20 | 31,195.00 | 31,361.20 | **31,118.80** | 31,243.23 | 31,328.00 | 31,296.00 | 0.32 | 0.57 |
| 38 | n275-s4 | 18,888.80 | 18,940.10 | 18,871.80 | 18,903.71 | **18,872.20** | **18,865.40** | -0.12 | -0.03 |
| 39 | n280-s100 | 31,886.20 | 32,197.80 | **31,722.20** | 31,959.69 | 33,436.80 | 33,302.80 | 4.44 | 4.98 |
| 40 | n284-s100 | 19,202.60 | 19,381.90 | **19,110.60** | 19,235.35 | 20,916.60 | 20,916.60 | 8.93 | 9.45 |
| 41 | n289-s20 | 71,639.00 | 71,748.80 | **71,434.60** | 71,542.48 | 72,467.40 | 72,011.60 | 0.52 | 0.81 |
| 42 | n294-s4 | 47,632.80 | 50,338.40 | 45,380.20 | 47,703.31 | **43,851.40** | **43,531.00** | -8.61 | -4.07 |
| 43 | n298-s100 | 31,502.60 | 31,715.40 | **31,246.20** | 31,485.55 | 31,721.80 | 31,550.00 | 0.15 | 0.97 |
| 44 | n303-s4 | 22,711.40 | 22,832.10 | 22,645.20 | 22,717.58 | **21,920.20** | **21,920.20** | -3.48 | -3.20 |
| 45 | n308-s20 | 23,324.00 | 23,563.80 | **22,928.60** | 23,185.37 | 25,271.60 | 25,167.00 | 7.90 | 9.76 |
| 46 | n313-s4 | 73,087.00 | 75,694.00 | **72,562.20** | 72,755.57 | 73,340.00 | 73,340.00 | 0.35 | 1.07 |
| 47 | n317-s100 | 58,312.80 | 58,438.30 | **58,279.40** | 58,380.68 | 59,046.00 | 59,046.00 | 1.26 | 1.32 |
| 48 | n322-s20 | 30,819.20 | 31,153.40 | 30,634.00 | 30,791.26 | **29,898.80** | **29,840.80** | -3.17 | -2.59 |
| 49 | n327-s4 | 26,801.80 | 26,987.70 | **26,489.60** | 26,763.68 | 27,369.20 | 27,369.20 | 2.12 | 3.32 |
| 50 | n331-s100 | 28,260.20 | 28,513.70 | **27,788.20** | 28,105.76 | 29,820.20 | 29,525.40 | 4.48 | 6.25 |
| 51 | n336-s20 | 122,533.40 | 123,044.30 | **121,921.00** | 122,361.76 | 126,065.00 | 126,065.00 | 2.88 | 3.40 |
| 52 | n344-s4 | 41,666.20 | 45,303.00 | 40,842.00 | 43,216.76 | **39,198.00** | **39,198.00** | -5.92 | -4.03 |
| 53 | n351-s20 | 26,021.00 | 26,112.10 | 25,919.00 | 26,010.77 | **25,389.20** | **25,389.20** | -2.43 | -2.04 |
| 54 | n359-s100 | 42,286.60 | 42,537.90 | **41,983.60** | 42,303.67 | 43,667.60 | 43,667.60 | 3.27 | 4.01 |
| 55 | n367-s20 | 22,101.60 | 22,322.00 | **21,656.80** | 21,996.53 | 23,550.40 | 23,550.40 | 6.56 | 8.74 |
| 56 | n376-s100 | 111,476.20 | 111,893.70 | **111,264.00** | 111,517.40 | 112,156.60 | 111,707.00 | 0.21 | 0.40 |
| 57 | n384-s4 | **45,470.40** | 45,656.70 | 45,304.80 | 45,441.98 | 45,818.40 | 45,730.80 | 0.57 | 0.94 |
| 58 | n393-s100 | 36,519.40 | 36,832.40 | **36,286.60** | 36,552.43 | 36,545.40 | 36,414.40 | -0.29 | 0.35 |
| 59 | n401-s20 | 53,452.00 | 53,738.00 | **53,338.80** | 53,495.55 | 55,107.40 | 54,984.80 | 2.87 | 3.09 |
| 60 | n411-s4 | 18,803.60 | 19,278.40 | **18,603.60** | 18,770.09 | 19,384.80 | 19,066.00 | 1.40 | 2.49 |
| 61 | n420-s4 | 93,630.60 | 95,750.60 | 90,620.09 | 91,132.09 | **90,654.80** | **89,791.20** | -4.10 | -0.91 |
| 62 | n429-s100 | 56,636.00 | 56,865.50 | **56,307.80** | 56,494.73 | 56,918.60 | 56,918.60 | 0.50 | 1.08 |
| 63 | n439-s20 | 35,016.40 | 35,549.20 | **35,009.60** | 35,294.73 | 36,049.00 | 36,037.80 | 2.92 | 2.94 |
| 64 | n449-s100 | 41,570.80 | 41,953.50 | **40,982.20** | 41,412.53 | 44,955.20 | 44,955.20 | 8.14 | 9.69 |
| 65 | n459-s20 | 24,685.40 | 25,046.40 | 24,187.20 | 24,520.72 | **23,607.80** | **23,535.80** | -4.66 | -2.69 |
| 66 | n469-s4 | 189,868.80 | 190,409.40 | **189,426.60** | 189,656.37 | 192,517.40 | 191,794.60 | 1.01 | 1.25 |
| 67 | n480-s4 | 89,458.20 | 97,438.80 | 86,695.20 | 87,876.17 | **86,680.60** | **86,680.60** | -3.10 | -0.02 |
| 68 | n491-s20 | 45,883.40 | 46,023.20 | **45,679.60** | 45,806.53 | 47,003.60 | 47,003.60 | 2.44 | 2.90 |
| 69 | n502-s100 | 54,261.80 | 54,480.60 | **54,200.00** | 54,327.01 | 54,763.60 | 54,741.80 | 0.88 | 1.00 |
| 70 | n513-s100 | 24,472.40 | 24,686.70 | **24,326.80** | 24,460.20 | 25,588.00 | 25,546.60 | 4.39 | 5.01 |
| 71 | n524-s4 | 131,259.60 | 133,072.80 | 128,261.40 | 129,852.68 | **127,237.40** | **126,892.00** | -3.33 | -1.07 |
| 72 | n536-s20 | 71,191.00 | 71,373.20 | **70,945.80** | 71,130.96 | 72,429.60 | 72,429.60 | 1.74 | 2.09 |
| 73 | n548-s100 | 62,789.20 | 63,100.80 | **62,343.00** | 62,610.65 | 64,161.20 | 64,132.80 | 2.14 | 2.87 |
| 74 | n561-s4 | 51,781.60 | 52,142.10 | 51,511.60 | 51,764.28 | **43,451.80** | **43,451.80** | -16.09 | -15.65 |
| 75 | n573-s20 | 39,521.00 | 39,728.10 | **39,401.00** | 39,515.33 | 41,804.00 | 41,804.00 | 5.78 | 6.10 |
| 76 | n586-s20 | 144,678.00 | 145,081.50 | **144,269.80** | 144,430.92 | 146,444.40 | 146,444.40 | 1.22 | 1.51 |
| 77 | n599-s4 | 96,209.40 | 96,508.80 | **95,793.60** | 95,975.48 | 97,032.60 | 97,032.60 | 0.86 | 1.29 |
| 78 | n613-s100 | 53,891.00 | 54,242.50 | **53,113.40** | 53,516.76 | 56,100.60 | 56,100.60 | 4.10 | 5.62 |
| 79 | n627-s4 | 59,217.40 | 60,552.00 | **58,026.20** | 58,944.11 | 61,316.80 | 61,316.80 | 3.55 | 5.67 |
| 80 | n641-s20 | 56,630.00 | 56,985.90 | **55,939.40** | 56,207.74 | 62,548.40 | 62,548.40 | 10.45 | 11.81 |
| 81 | n655-s100 | 73,186.60 | 73,269.50 | **72,824.00** | 72,968.71 | 73,755.60 | 73,755.60 | 0.78 | 1.28 |
| 82 | n670-s4 | 98,178.20 | 98,832.00 | **97,818.20** | 98,238.34 | 100,137.20 | 100,137.20 | 2.00 | 2.37 |
| 83 | n685-s20 | 63,970.60 | 64,405.40 | **63,284.20** | 63,740.19 | 64,457.60 | 64,457.60 | 0.76 | 1.85 |

Table 8: Our best results (1,200 seconds) for the instances given in Toffolo et al (2018)

| # | Testinstance | T18 (600 sec.) | | T18 (1,200 sec.) | | Prop. algorithm 75% | | Gap | Gap |
|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Avg. Sol. | VNS | SP | Best 600 sec. | Best 1,200 sec. |
| 84 | n701-s100 | 68,307.80 | 69,105.00 | **67,454.60** | 68,172.87 | 73,256.40 | 73,256.40 | 7.24 | 8.60 |
| 85 | n716-s20 | 38,738.60 | 39,078.70 | **38,168.20** | 38,450.13 | 39,746.80 | 39,746.80 | 2.60 | 4.14 |
| 86 | n733-s4 | 141,618.40 | 155,822.30 | **135,869.80** | 146,352.27 | 136,116.00 | 135,908.40 | -4.03 | 0.03 |
| 87 | n749-s100 | 55,881.00 | 56,116.00 | **55,502.80** | 55,723.82 | 58,382.00 | 58,382.00 | 4.48 | 5.19 |
| 88 | n766-s100 | 105,462.80 | 106,062.00 | **104,365.80** | 104,980.10 | 109,156.60 | 109,156.60 | 3.50 | 4.59 |
| 89 | n783-s4 | 61,231.80 | 61,655.60 | **60,608.80** | 60,879.23 | 66,574.60 | 66,574.60 | 8.73 | 9.84 |
| 90 | n801-s20 | 60,632.80 | 61,576.70 | **59,891.00** | 60,179.16 | 68,358.00 | 68,232.60 | 12.53 | 13.93 |
| 91 | n819-s100 | 124,521.60 | 125,217.10 | **123,375.80** | 124,092.90 | 125,916.60 | 125,916.60 | 1.12 | 2.06 |
| 92 | n837-s4 | 133,712.00 | 134,346.00 | **132,397.20** | 132,976.66 | 136,319.60 | 136,319.60 | 1.95 | 2.96 |
| 93 | n856-s20 | 87,743.80 | 88,029.30 | **85,840.20** | 86,955.92 | 85,948.60 | 85,948.60 | -2.05 | 0.13 |
| 94 | n876-s20 | 90,187.60 | 90,740.00 | **89,734.80** | 89,977.46 | 95,377.00 | 95,377.00 | 5.75 | 6.29 |
| 95 | n895-s4 | 41,907.60 | 42,107.50 | **41,656.80** | 41,747.53 | 47,607.60 | 47,603.20 | 13.59 | 14.27 |
| 96 | n916-s100 | 240,702.40 | 241,110.20 | **239,844.80** | 240,140.84 | 243,422.20 | 243,422.20 | 1.13 | 1.49 |
| 97 | n936-s4 | 154,993.20 | 158,350.00 | 129,633.40 | 133,332.20 | **126,264.60** | **126,264.60** | -18.54 | -2.60 |
| 98 | n957-s100 | 73,133.60 | 73,653.70 | **72,555.80** | 72,926.08 | 76,364.20 | 76,364.20 | 4.42 | 5.25 |
| 99 | n979-s20 | 107,932.20 | 109,252.50 | **106,815.60** | 107,572.13 | 117,320.40 | 117,320.40 | 8.70 | 9.83 |
| 100 | n1001-s4 | 76,172.80 | 76,996.90 | **74,188.80** | 75,520.17 | 79,429.40 | 79,429.40 | 4.28 | 7.06 |
| | **Avg. Gap** | | | | | | | 0.88 | 1.93 |

#### *4.2.1 VeRoLog instances*

Table 9 illustrates the results found by VNS-SP on the SB-VRP instances introduced in Heid et al (2014) and a comparison with those reported in Absi et al (2015) (*A15*), Huber and Geiger (2017) (*H17*), Miranda-Bront et al (2017) (*M17*), Todosijević et al (2016) (*T16*) and Toffolo et al (2018) (*T18*). Each of those solution approaches was timed to 10 minutes. In addition, the computational study of Absi et al (2015) includes experiments with 20 minutes. Note that no solutions were provided by Absi et al (2015) and Todosijević et al (2016) for six benchmark instances.

*Prop. algorithm 75%* as well as *Prop. algorithm 25%* matched the best known solution in the *small - all without trailer* instance and managed to improve 6 best solutions. The average gap to the best solution was 1.42%. VNS-SP is beneficial for instances #13 - #15. Our matheuristic consistently delivers higher quality solutions than the other approaches (Absi et al, 2015; Huber and Geiger, 2017; Lum et al, 2015; Miranda-Bront et al, 2017; Todosijević et al, 2016).

The VNS by itself is capable of achieving, on average, competitive results in terms of solution quality when compared to the best results reported in Huber and Geiger (2017); Toffolo et al (2018). However, for the *final random normal, final random all without trailer* and *final random all with trailer* the results of VNS-SP are much better than all of the other approaches having the same computational time. The major factor that influences these savings for the objective function values seems to be the customer demands. This can especially be validated when the *final random normal* is compared with the *final normal* instance. For both instances, the x- and y-coordinates are the same as well as the distance and time values for each arc. The locations seem to have a minor influence. In addition, the set partitioning formulation seems beneficial for these instances: *final random normal*, *final random all with trailer* and *final random all without trailer*. Also, the number of train and truck configurations is much higher for the *final random normal* than for the *final normal* instance, which allows the SP model to have a greater benefit on the solution quality. Comparing one solution of a *final random normal* with one of *final normal* instance shows that 1.2 times more trucks, 7.6 times more train and 15.7 times more subtours are utilized.

Table 10 reports the average values in comparison to the existing approaches. *Prop. algorithm 75%* obtained on average the best results for *final random all with trailer* and *final random all without trailer* instance. The *final random normal* instance was solved

Table 9: Our best results for the instances proposed by the organizers of the VeRoLog Solver Challenge (Heid et al, 2014)

| # | Instance | T18 | | H17 | M17 | T16 | A15 | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Best Sol. | Best Sol. | Best Sol. | VNS | SP | VNS | SP | VNS | SP | Best |
| 1 | small normal | 4,797.55 | 4,797.69 | 4,797.55 | 4,806.97 | 4,847.63 | 4,802.38 | **4,727.63** | **4,727.63** | **4,727.33** | **4,727.33** | **4,727.63** | **4,727.63** | -1.46 |
| 2 | small all without tr. | **4,839.64** | 4,858.53 | **4,839.64** | 4,855.62 | 5,249.18 | 4,981.70 | 4,896.48 | **4,839.64** | 4,896.48 | **4,839.64** | 4,913.39 | **4,839.64** | 0.00 |
| 3 | small all with tr. | 4,715.78 | 4,716.50 | 4,716.58 | 4,728.93 | 4,731.02 | 4,716.58 | **4,660.51** | **4,646.36** | **4,660.51** | **4,648.75** | **4,660.70** | **4,647.88** | -1.47 |
| 4 | medium normal | 7,803.54 | 7,818.29 | 7,795.98 | 7,942.22 | 7,834.78 | 7,810.93 | 7,862.89 | 7,852.03 | 7,855.34 | 7,845.13 | 7,815.12 | **7,791.96** | -0.05 |
| 5 | medium all without tr. | **7,980.01** | 8,007.11 | 7,982.76 | 8,169.69 | 8,382.80 | 8,058.89 | 8,130.45 | 8,078.47 | 8,066.58 | 8,060.70 | 8,040.23 | 8,040.23 | 0.75 |
| 6 | medium all with tr. | **7,708.63** | 7,745.80 | 7,734.61 | 7,847.30 | 7,754.39 | 7,763.13 | 7,883.26 | 7,807.50 | 7,823.61 | 7,768.52 | 7,761.30 | 7,758.79 | 0.65 |
| 7 | large1 normal | **19,985.88** | 20,039.92 | 20,298.82 | 20,738.50 | 20,496.40 | 20,760.30 | 20,895.07 | 20,895.07 | 20,733.03 | 20,733.03 | 20,685.51 | 20,685.51 | 3.50 |
| 8 | large1 all without tr. | **20,640.72** | 20,728.43 | 21,003.51 | 21,522.50 | 22,310.60 | 21,560.80 | 21,768.40 | 21,768.40 | 21,397.04 | 21,397.04 | 21,335.40 | 21,335.40 | 3.37 |
| 9 | large1 all with tr. | **19,806.75** | 19,851.04 | 20,058.99 | 20,516.70 | 20,066.40 | 20,495.70 | 20,672.87 | 20,672.87 | 20,527.85 | 20,527.85 | 20,280.56 | 20,280.56 | 2.39 |
| 10 | large2 normal | **24,800.16** | 24,859.76 | 25,069.86 | 25,894.40 | 25,443.20 | 25,529.50 | 25,954.74 | 25,954.74 | 25,704.64 | 25,704.64 | 25,635.90 | 25,635.90 | 3.37 |
| 11 | large2 all without tr. | **25,448.55** | 25,510.38 | 25,719.19 | 26,524.50 | 26,515.90 | 25,975.50 | 26,714.76 | 26,714.47 | 26,261.69 | 26,261.69 | 26,123.56 | 26,123.56 | 2.65 |
| 12 | large2 all with tr. | **24,402.67** | 24,475.56 | 24,767.63 | 25,573.20 | 24,965.10 | 25,021.70 | 25,589.44 | 25,555.22 | 25,283.61 | 25,283.61 | 25,223.09 | 25,223.09 | 3.36 |
| 13 | final random normal | 132,341.83 | 132,758.74 | 132,295.68 | 135,509.00 | - | - | 133,187.62 | **128,938.45** | 132,917.37 | **128,636.52** | **132,286.02** | **128,690.14** | -2.77 |
| 14 | final random all without tr. | 144,331.84 | 144,588.22 | 144,725.57 | 152,587.00 | - | - | 145,668.08 | **143,691.91** | 145,405.31 | **143,661.83** | 145,081.11 | **143,631.74** | -0.49 |
| 15 | final random all with tr. | 129,049.18 | 129,356.15 | 129,257.44 | 131,445.00 | - | - | 129,726.69 | **125,258.54** | 129,757.06 | **125,043.80** | 129,420.67 | **124,760.48** | -3.32 |
| 16 | final normal | **33,927.04** | 34,254.51 | 34,649.78 | 36,305.80 | - | - | 36,812.18 | 36,812.18 | 36,282.13 | 36,123.44 | 35,814.32 | 35,814.32 | 5.56 |
| 17 | final all without tr. | **36,347.28** | 36,574.23 | 36,814.84 | 38,826.20 | - | - | 38,752.64 | 38,752.64 | 38,256.74 | 38,256.74 | 37,830.57 | 37,830.57 | 4.08 |
| 18 | final all with tr. | **33,014.03** | 33,118.08 | 33,753.48 | 34,997.90 | - | - | 35,284.01 | 35,282.98 | 35,040.95 | 35,040.95 | 34,787.57 | 34,787.57 | 5.37 |
| | **Avg. Gap** | | | | | | | | | | | | | 1.42 |

Table 10: Our average results for the instances proposed by the organizers of the VeRoLog Solver Challenge (Heid et al, 2014)

| # | Instance | T18 | | H17 | M17 | T16 | A15 | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Best Sol. | Best Sol. | Best Sol. | VNS | SP | VNS | SP | VNS | SP | Best |
| 1 | small normal | 4,797.55 | 4,797.69 | 4,797.55 | 4,806.97 | 4,847.63 | 4,802.38 | **4,740.67** | **4,732.50** | **4,738.03** | **4,729.88** | **4,747.13** | **4,731.94** | -1.41 |
| 2 | small all without tr. | **4,839.64** | 4,858.53 | 4,839.64 | 4,855.62 | 5,249.18 | 4,981.70 | 4,916.41 | 4,854.24 | 4,912.30 | 4,846.71 | 4,918.13 | 4,849.74 | 0.15 |
| 3 | small all with tr. | 4,715.78 | 4,716.50 | 4,716.58 | 4,728.93 | 4,731.02 | 4,716.58 | **4,664.33** | **4,657.50** | **4,661.82** | **4,655.89** | **4,662.77** | **4,658.22** | -1.27 |
| 4 | medium normal | 7,803.54 | 7,818.29 | **7,795.98** | 7,942.22 | 7,834.78 | 7,810.93 | 7,942.13 | 7,903.06 | 7,931.69 | 7,890.85 | 7,898.59 | 7,877.16 | 1.04 |
| 5 | medium all without tr. | **7,980.01** | 8,007.11 | 7,982.76 | 8,169.69 | 8,382.80 | 8,058.89 | 8,192.09 | 8,117.74 | 8,140.52 | 8,112.04 | 8,136.01 | 8,103.89 | 1.55 |
| 6 | medium all with tr. | **7,708.63** | 7,745.80 | 7,734.61 | 7,847.30 | 7,754.39 | 7,763.13 | 7,908.37 | 7,851.75 | 7,856.40 | 7,812.74 | 7,818.66 | 7,779.36 | 0.92 |
| 7 | large1 normal | **19,985.88** | 20,039.92 | 20,298.82 | 20,738.50 | 20,496.40 | 20,760.30 | 21,113.75 | 21,113.37 | 20,830.95 | 20,829.83 | 20,782.36 | 20,782.36 | 3.99 |
| 8 | large1 all without tr. | **20,640.72** | 20,728.43 | 21,003.51 | 21,522.50 | 22,310.60 | 21,560.80 | 21,913.51 | 21,911.88 | 21,616.19 | 21,616.00 | 21,527.81 | 21,527.81 | 4.30 |
| 9 | large1 all with tr. | **19,806.75** | 19,851.04 | 20,058.99 | 20,516.70 | 20,066.40 | 20,495.70 | 20,900.15 | 20,899.58 | 20,637.98 | 20,637.98 | 20,528.49 | 20,528.49 | 3.64 |
| 10 | large2 normal | **24,800.16** | 24,859.76 | 25,069.86 | 25,894.40 | 25,443.20 | 25,529.50 | 26,153.68 | 26,138.02 | 25,825.57 | 25,824.66 | 25,730.77 | 25,730.77 | 3.75 |
| 11 | large2 all without tr. | **25,448.55** | 25,510.38 | 25,719.19 | 26,524.50 | 26,515.90 | 25,975.50 | 26,849.91 | 26,848.45 | 26,447.66 | 26,445.73 | 26,334.13 | 26,334.13 | 3.48 |
| 12 | large2 all with tr. | **24,402.67** | 24,475.56 | 24,767.63 | 25,573.20 | 24,965.10 | 25,021.70 | 25,705.12 | 25,698.18 | 25,454.57 | 25,454.57 | 25,339.87 | 25,339.87 | 3.84 |
| 13 | final random normal | 132,341.83 | 132,758.74 | 132,295.68 | 135,509.00 | - | - | 133,675.44 | **129,225.91** | 133,205.93 | **129,043.16** | 132,905.46 | **129,089.95** | -2.46 |
| 14 | final random all without tr. | 144,331.84 | 144,588.22 | 144,725.57 | 152,587.00 | - | - | 146,059.86 | **143,875.74** | 145,580.46 | **143,812.85** | 145,293.78 | **143,703.28** | -0.44 |
| 15 | final random all with tr. | 129,049.18 | 129,356.15 | 129,257.44 | 131,445.00 | - | - | 130,655.43 | **125,672.23** | 130,129.51 | **125,556.95** | 130,020.01 | **125,329.76** | -2.88 |
| 16 | final normal | **33,927.04** | 34,254.51 | 34,649.78 | 36,305.80 | - | - | 37,194.37 | 37,175.98 | 36,734.56 | 36,702.48 | 36,281.40 | 36,281.40 | 6.94 |
| 17 | final all without tr. | **36,347.28** | 36,574.23 | 36,814.84 | 38,826.20 | | - | 39,065.24 | 39,059.54 | 38,536.09 | 38,534.74 | 38,423.77 | 38,423.77 | 5.71 |
| 18 | final all with tr. | **33,014.03** | 33,118.08 | 33,753.48 | 34,997.90 | - | - | 35,938.99 | 35,937.25 | 35,540.04 | 35,540.04 | 35,074.98 | 35,074.98 | 6.24 |
| | **Avg. Gap** | | | | | | | | | | | | | 2.06 |

Table 11: Best results (1,200 seconds) for the VeRoLog instances

| # | Testinstance | T18 | H17 | M17 | T16 | A15 | Prop. algorithm 75% | | Gap |
|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Best Sol. | Best Sol. | Best Sol. | Best Sol. | VNS | SP | Best |
| 1 | small normal | 4,797.55 | 4,797.55 | 4,806.97 | 4,847.63 | 4,802.38 | **4,727.63** | **4,727.33** | -1.46 |
| 2 | small all without tr. | **4,839.64** | 4,839.64 | 4,855.62 | 5,249.18 | 4,981.70 | 4,913.39 | **4,839.64** | 0.00 |
| 3 | small all with tr. | 4,715.78 | 4,716.58 | 4,728.93 | 4,731.02 | 4,716.58 | **4,660.51** | **4,651.92** | -1.35 |
| 4 | medium normal | 7,803.54 | **7,795.98** | 7,942.22 | 7,834.78 | 7,810.93 | 7,853.23 | 7,818.22 | 0.29 |
| 5 | medium all without tr. | **7,980.01** | 7,982.76 | 8,169.69 | 8,382.80 | 8,058.89 | 8,044.49 | 8,022.68 | 0.53 |
| 6 | medium all with tr. | **7,708.63** | 7,734.61 | 7,847.30 | 7,754.39 | 7,763.13 | 7,759.82 | 7,736.78 | 0.37 |
| 7 | large1 normal | **19,985.88** | 20,298.82 | 20,738.50 | 20,496.40 | 20,760.30 | 20,469.92 | 20,469.92 | 2.42 |
| 8 | large1 all without tr. | **20,640.72** | 21,003.51 | 21,522.50 | 22,310.60 | 21,560.80 | 21,192.00 | 21,192.00 | 2.67 |
| 9 | large1 all with tr. | **19,806.75** | 20,058.99 | 20,516.70 | 20,066.40 | 20,495.70 | 20,256.43 | 20,256.43 | 2.27 |
| 10 | large2 normal | **24,800.16** | 25,069.86 | 25,894.40 | 25,443.20 | 25,529.50 | 25,428.72 | 25,416.39 | 2.48 |
| 11 | large2 all without tr. | **25,448.55** | 25,719.19 | 26,524.50 | 26,515.90 | 25,975.50 | 25,939.03 | 25,939.03 | 1.93 |
| 12 | large2 all with tr. | **24,402.67** | 24,767.63 | 25,573.20 | 24,965.10 | 25,021.70 | 25,001.83 | 24,985.95 | 2.39 |
| 13 | final rand. normal | 132,341.83 | 132,295.68 | 135,509.00 | - | - | **131,776.00** | **128,735.96** | -2.69 |
| 14 | final rand. all without tr. | 144,331.84 | 144,725.57 | 152,587.00 | - | - | 145,045.71 | **143,652.39** | -0.47 |
| 15 | final rand. all with tr. | 129,049.18 | 129,257.44 | 131,445.00 | - | - | **128,949.97** | **125,028.31** | -3.12 |
| 16 | final normal | **33,927.04** | 34,649.78 | 36,305.80 | - | - | 35,409.03 | 35,409.03 | 4.37 |
| 17 | final all without tr. | **36,347.28** | 36,814.84 | 38,826.20 | - | - | 37,694.38 | 37,694.38 | 3.71 |
| 18 | final all with tr. | **33,014.03** | 33,753.48 | 34,997.90 | - | - | 34,350.58 | 34,350.58 | 4.05 |
| | **Avg. Gap** | | | | | | | | 1.02 |

best by *Prop. algorithm 50%*. The average gap between the best average solutions and best solutions was only 2.06%.

*4.2.2 Absi et al. instances*

Our best results are given in Table 12 for the 20 instances provided in Absi et al (2015) and compared to the results of Absi et al (2015) and Huber and Geiger (2017). In total, nine new best known solutions can be found by the VNS-SP and one solution can be matched for the *C3 small* data set. The SP formulation especially gives an advantage for the *C1 medium*, *C2 large2*, *C2 medium*, *C4 medium* and *C4 small* since based on the pool of routes provided by the VNS a better selection can be established.

On average, we could improve the best solutions by 0.266%. With respect to the average results reported in Table 13, the VNS-SP was able to improve the best results for four benchmark instances. Also, the average gap was competitive with 0.142%.

The study of Absi et al (2015) included experiments with a termination criterion of 20 minutes. Thus, we provide our best results (out of 10 runs) in Table 14 for the variant *Prop. algorithm 75%*. With this extended running time the VNS-SP can further improve the best results. In particular, 11 results can be outperformed, resulting in an average gap of 0.442%.

## 5 Conclusions

This paper has introduced a VNS-SP for the SB-VRP, an important generalization of the CVRP. This matheuristic is based on a pool of routes to enhance the quality of the solutions. Moreover, several ideas were presented to update the pool. One of our findings is that it is important to include routes from an early stage of the search. In detail the allocation of computational time to the SP formulation was analyzed with three different variants for two instance classes containing 138 data sets in total.

Our computational experiments and analysis confirm that combining VNS with a set partitioning formulation can result in substantial benefits, but also show that for some instances the VNS alone achieves better results. In particular, the VNS-SP achieved high quality solutions for several data sets. In total, 42 new best solutions were found, where the maximum improvement was 18.54%. For example, all the existing

Table 12: Our best results for the instances provided by Absi et al (2015)

| # | Instance | A15 | | H17 | | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap Best |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Avg. Sol. | VNS | SP | VNS | SP | VNS | SP | |
| 1 | C1 large1 | 20,008.50 | 20,246.73 | 18,812.41 | 19,031.76 | 19,349.54 | 19,348.39 | 18,807.91 | **18,807.91** | **18,796.24** | **18,796.24** | -0.086 |
| 2 | C1 large2 | 21,084.30 | 21,328.52 | **20,894.22** | 21,153.84 | 21,604.43 | 21,604.39 | 20,934.24 | 20,931.12 | 20,909.40 | 20,909.40 | 0.073 |
| 3 | C1 medium | 7,411.30 | 7,532.70 | 7,292.13 | 7,396.62 | 7,385.77 | 7,351.96 | 7,292.42 | **7,251.65** | 7,267.64 | **7,259.13** | -0.555 |
| 4 | C1 small | 4,292.07 | 4,302.80 | 4,309.01 | 4,357.08 | **4,204.33** | **4,202.33** | **4,204.33** | **4,195.49** | **4,195.49** | **4,195.49** | -2.250 |
| 5 | C2 large1 | **37,954.90** | 38,031.15 | 38,273.61 | 38,409.30 | 38,888.29 | 38,056.26 | 38,515.61 | 38,038.76 | 38,411.47 | 37,981.19 | 0.069 |
| 6 | C2 large2 | 50,052.80 | 50,076.43 | 50,093.70 | 50,314.54 | 50,575.51 | 50,077.98 | 50,295.21 | 50,072.64 | 50,143.35 | **49,995.18** | -0.197 |
| 7 | C2 medium | 14,450.50 | 14,457.48 | 14,447.51 | 14,490.59 | 14,504.48 | **14,445.62** | 14,487.46 | **14,436.73** | 14,480.35 | **14,437.99** | -0.075 |
| 8 | C2 small | **3,437.63** | 3,444.68 | 3,479.94 | 3,479.94 | 3,442.30 | 3,442.30 | 3,442.30 | 3,442.30 | 3,442.30 | 3,442.30 | 0.136 |
| 9 | C3 large1 | 17,606.80 | 17,776.52 | **17,573.49** | 17,715.46 | 18,399.11 | 18,399.11 | 17,815.61 | 17,778.86 | 17,700.72 | 17,695.30 | 0.693 |
| 10 | C3 large2 | **22,806.50** | 22,874.95 | 22,835.79 | 23,055.37 | 23,794.17 | 23,658.94 | 23,146.36 | 23,128.40 | 23,045.57 | 23,045.57 | 1.048 |
| 11 | C3 medium | **6,809.87** | 6,820.14 | 6,819.36 | 6,879.43 | 6,889.39 | 6,887.67 | 6,880.23 | 6,877.32 | 6,867.08 | 6,867.08 | 0.840 |
| 12 | C3 small | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | 0.000 |
| 13 | C4 large1 | 30,183.10 | 30,409.00 | 28,890.85 | 29,071.78 | 29,597.07 | 29,513.12 | 29,089.08 | 29,081.75 | **28,876.29** | **28,876.29** | -0.050 |
| 14 | C4 large2 | 33,601.20 | 33,835.42 | 31,596.90 | 31,770.11 | 32,299.85 | 32,299.85 | 31,759.78 | **31,555.74** | 31,731.10 | 31,731.10 | -0.130 |
| 15 | C4 medium | 11,015.70 | 11,072.40 | 10,893.66 | 10,950.33 | 10,979.55 | **10,845.78** | 10,923.83 | **10,829.99** | 10,939.34 | **10,837.73** | -0.513 |
| 16 | C4 small | 6,950.81 | 6,981.30 | 6,934.71 | 6,934.71 | **6,785.73** | **6,566.29** | **6,773.65** | **6,566.29** | **6,778.66** | **6,566.29** | -5.313 |
| 17 | C5 large1 | **37,338.10** | 37,410.13 | 37,740.84 | 37,878.22 | 38,504.46 | 37,511.74 | 38,006.17 | 37,561.83 | 37,809.96 | 37,596.82 | 0.465 |
| 18 | C5 large2 | **50,054.60** | 50,066.77 | 50,139.53 | 50,313.94 | 50,682.73 | 50,087.56 | 50,303.00 | 50,056.51 | 50,176.32 | 50,066.25 | 0.004 |
| 19 | C5 medium | **14,329.80** | 14,335.62 | 14,338.43 | 14,351.06 | 14,351.02 | 14,341.43 | 14,351.11 | 14,334.86 | 14,348.51 | 14,338.45 | 0.035 |
| 20 | C5 small | **3,222.84** | 3,222.84 | 3,272.11 | 3,272.11 | 3,238.51 | 3,238.51 | 3,238.51 | 3,238.51 | 3,238.51 | 3,238.51 | 0.486 |
| | **Avg. Gap** | | | | | | | | | | | -0.266 |

Table 13: Our average results for the instances provided by Absi et al (2015)

| # | Instance | A15 | | H17 | | Prop. algorithm 25% | | Prop. algorithm 50% | | Prop. algorithm 75% | | Gap Best |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Avg. Sol. | VNS | SP | VNS | SP | VNS | SP | |
| 1 | C1 large1 | 20,008.50 | 20,246.73 | **18,812.41** | 19,031.76 | 19,352.43 | 19,351.51 | 19,021.08 | 19,020.49 | 18,996.74 | 18,993.70 | 0.964 |
| 2 | C1 large2 | 21,084.30 | 21,328.52 | **20,894.22** | 21,153.84 | 21,623.55 | 21,623.54 | 21,208.78 | 21,202.21 | 21,137.22 | 21,136.14 | 1.158 |
| 3 | C1 medium | 7,411.30 | 7,532.70 | **7,292.13** | 7,396.62 | 7,412.63 | 7,385.76 | 7,355.30 | 7,342.28 | 7,352.47 | 7,334.15 | 0.576 |
| 4 | C1 small | 4,292.07 | 4,302.80 | 4,309.01 | 4,357.08 | **4,206.14** | **4,203.71** | **4,220.76** | **4,208.13** | **4,205.39** | **4,198.02** | -2.191 |
| 5 | C2 large1 | **37,954.90** | 38,031.15 | 38,273.61 | 38,409.30 | 38,888.29 | 38,106.74 | 38,593.28 | 38,161.81 | 38,535.94 | 38,189.77 | 0.400 |
| 6 | C2 large2 | **50,052.80** | 50,076.43 | 50,093.70 | 50,314.54 | 50,655.68 | 50,094.92 | 50,338.10 | 50,091.94 | 50,257.04 | 50,070.85 | 0.036 |
| 7 | C2 medium | 14,450.50 | 14,457.48 | 14,447.51 | 14,490.59 | 14,505.32 | **14,445.62** | 14,509.16 | 14,448.07 | 14,497.01 | 14,448.37 | -0.013 |
| 8 | C2 small | **3,437.63** | 3,444.68 | 3,479.94 | 3,479.94 | 3,442.30 | 3,442.30 | 3,442.30 | 3,442.30 | 3,442.30 | 3,442.30 | 0.136 |
| 9 | C3 large1 | 17,606.80 | 17,776.52 | **17,573.49** | 17,715.46 | 18,446.43 | 18,433.76 | 17,966.08 | 17,962.35 | 17,857.68 | 17,857.14 | 1.614 |
| 10 | C3 large2 | **22,806.50** | 22,874.95 | 22,835.79 | 23,055.37 | 23,818.94 | 23,693.35 | 23,385.78 | 23,327.27 | 23,152.85 | 23,134.22 | 1.437 |
| 11 | C3 medium | **6,809.87** | 6,820.14 | 6,819.36 | 6,879.43 | 6,896.30 | 6,894.28 | 6,909.83 | 6,899.27 | 6,904.51 | 6,899.72 | 1.239 |
| 12 | C3 small | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | 0.000 |
| 13 | C4 large1 | 30,183.10 | 30,409.00 | **28,890.85** | 29,071.78 | 29,599.18 | 29,525.86 | 29,210.16 | 29,182.00 | 29,085.00 | 29,056.91 | 0.575 |
| 14 | C4 large2 | 33,601.20 | 33,835.42 | **31,596.90** | 31,770.11 | 32,656.85 | 32,600.32 | 32,037.63 | 31,976.38 | 31,956.15 | 31,920.11 | 1.023 |
| 15 | C4 medium | 11,015.70 | 11,072.40 | 10,893.66 | 10,950.33 | 11,036.05 | 10,901.81 | 10,963.74 | **10,868.01** | 10,992.78 | 10,899.11 | -0.235 |
| 16 | C4 small | 6,950.81 | 6,981.30 | 6,934.71 | 6,934.71 | **6,786.67** | **6,566.29** | **6,785.89** | **6,566.29** | **6,786.83** | **6,566.29** | -5.313 |
| 17 | C5 large1 | **37,338.10** | 37,410.13 | 37,740.84 | 37,878.22 | 38,504.46 | 37,625.85 | 38,115.70 | 37,679.23 | 38,038.79 | 37,694.41 | 0.771 |
| 18 | C5 large2 | **50,054.60** | 50,066.77 | 50,139.53 | 50,313.94 | 50,705.85 | 50,099.60 | 50,395.70 | 50,097.80 | 50,345.10 | 50,083.12 | 0.057 |
| 19 | C5 medium | **14,329.80** | 14,335.62 | 14,338.43 | 14,351.06 | 14,354.54 | 14,349.37 | 14,353.76 | 14,346.37 | 14,353.04 | 14,346.93 | 0.116 |
| 20 | C5 small | **3,222.84** | **3,222.84** | 3,272.11 | 3,272.11 | 3,238.51 | 3,238.51 | 3,238.51 | 3,238.51 | 3,238.51 | 3,238.51 | 0.486 |
| | **Avg. Gap** | | | | | | | | | | | 0.142 |

Table 14: Our best results (1,200 seconds) for the instances provided in Absi et al (2015)

| # | Instance | A15 | | H17 | | Prop. algorithm 75% | | Gap Best |
|---|---|---|---|---|---|---|---|---|
| | | Best Sol. | Avg. Sol. | Best Sol. | Avg. Sol. | VNS | SP | |
| 1 | C1 large1 | 20,008.50 | 20,246.73 | 18,812.41 | 19,031.76 | **18,753.31** | **18,753.31** | -0.314 |
| 2 | C1 large2 | 21,084.30 | 21,328.52 | 20,894.22 | 21,153.84 | **20,877.48** | **20,877.48** | -0.080 |
| 3 | C1 medium | 7,411.30 | 7,532.70 | 7,292.13 | 7,396.62 | **7,193.22** | **7,192.13** | -1.371 |
| 4 | C1 small | 4,292.07 | 4,302.80 | 4,309.01 | 4,357.08 | **4,195.49** | **4,195.49** | -2.250 |
| 5 | C2 large1 | 37,954.90 | 38,031.15 | 38,273.61 | 38,409.30 | 38,266.53 | **37,776.58** | -0.470 |
| 6 | C2 large2 | **50,052.80** | 50,076.43 | 50,093.70 | 50,314.54 | 50,134.30 | 50,054.42 | 0.003 |
| 7 | C2 medium | 14,450.50 | 14,457.48 | 14,447.51 | 14,490.59 | 14,477.70 | **14,434.84** | -0.088 |
| 8 | C2 small | **3,437.63** | 3,444.68 | 3,479.94 | 3,479.94 | 3,442.30 | 3,442.30 | 0.136 |
| 9 | C3 large1 | 17,606.80 | 17,776.52 | **17,573.49** | 17,715.46 | 17,667.71 | 17,667.71 | 0.536 |
| 10 | C3 large2 | **22,806.50** | 22,874.95 | 22,835.79 | 23,055.37 | 22,908.72 | 22,908.72 | 0.448 |
| 11 | C3 medium | **6,809.87** | 6,820.14 | 6,819.36 | 6,879.43 | 6,864.76 | 6,831.03 | 0.311 |
| 12 | C3 small | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | **3,975.87** | 0.000 |
| 13 | C4 large1 | 30,183.10 | 30,409.00 | 28,890.85 | 29,071.78 | **28,883.79** | **28,854.60** | -0.125 |
| 14 | C4 large2 | 33,601.20 | 33,835.42 | 31,596.90 | 31,770.11 | **31,553.81** | **31,553.81** | -0.136 |
| 15 | C4 medium | 11,015.70 | 11,072.40 | 10,893.66 | 10,950.33 | **10,842.66** | **10,790.51** | -0.947 |
| 16 | C4 small | 6,950.81 | 6,981.30 | 6,934.71 | 6,934.71 | **6,769.32** | **6,566.29** | -5.313 |
| 17 | C5 large1 | **37,338.10** | 37,410.13 | 37,740.84 | 37,878.22 | 37,752.54 | 37,511.21 | 0.464 |
| 18 | C5 large2 | 50,054.60 | 50,066.77 | 50,139.53 | 50,313.94 | 50,145.33 | **49,972.53** | -0.164 |
| 19 | C5 medium | **14,329.80** | 14,335.62 | 14,338.43 | 14,351.06 | 14,349.19 | 14,333.92 | 0.029 |
| 20 | C5 small | **3,222.84** | **3,222.84** | 3,272.11 | 3,272.11 | 3,238.51 | 3,238.51 | 0.486 |
| | **Avg. Gap** | | | | | | | -0.442 |

approaches were outperformed for the *final random normal*, *final random all with trailer* and *final random all without trailer* instance. The partitioning formulation is capable of enhancing the route diversification and choosing good route combinations.

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