Facility Location with Modular Capacity under Demand Uncertainty: An Industrial Case Study

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Abstract

We investigate a facility location problem with modular capacity under demand uncertainty arising at Hydro-Québec, the largest public utility in Canada. We propose a mathematical model to locate the facilities, determine the capacity levels, and compute the number of service teams needed in each facility to satisfy the service demand of all customers. We implement a two-stage stochastic optimization framework to address the uncertainty in customer demand. We present a traditional scenario-based stochastic optimization method and a linear decision rule (LDR)-based solution method. We highlight the significant reduction in the computational time provided by the latter, particularly for large instances. These gains in computational performance come at the expense of probable overly robust location decisions. To manage this possible outcome, we adapt a feedback concept found in process system controllers, and we develop two LDR robust trade-off heuristic algorithms that combine a linear decision method with the feedback mechanism. The benefits are empirically illustrated via numerical experiments and validated on an industrial case study in which we achieve a 1.37% to 5.20% reduction in average total costs.

Keywords: stochastic programming, decision rules, facility location, demand uncertainty

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1. Introduction

Facility location problems (FLPs) are central in location science (Laporte et al., 2019). Their aim is to determine the number and location of facilities required to fulfill the demand of a set of customers at minimal cost. This is achieved by optimizing the strategic facility location decisions together with the tactical decisions regarding the capacity of facilities and the allocation of customers to facilities.

In this work, we address a facility location problem with modular capacity and uncertain demand arising at Hydro-Québec, the largest public utility in Canada. Hydro-Québec is responsible for electricity generation, transmission, and distribution in the province of Québec in Canada. Its main source of production is based on renewable energies through hydroelectric power plants and the company works constantly to support the development of clean energy technologies (Hydro-Québec, 2020). It manages 63 hydroelectric power stations and 24 thermal power stations, with a total production capacity of 37,310 MW. It oversees a massive transmission and distribution network serving 4.3 million customers (Hydro-Québec, 2017, 2018).

To maintain its operations, the company relies on a network of service centers housing a number of service teams. The type of service provided by a team depends on their assigned functionality (e.g., service installation or distribution lines maintenance) and customer type (e.g., individual or industrial). The locations and capacities of service centers were decided over time in a reactive manner in response to increases in customer demands. The aim of this work is to propose a systematic approach to locate the service centers (i.e., facilities) and determine their capacities at the minimum total expected cost under uncertain demand.

1.1. Literature review

Capacitated facility location problems (CFLPs) are a specific class within the FLP family. As the name suggests, the potential facilities in a CFLP have a predefined capacity. The capacity decisions vary between strategic, e.g., maximum capacity, and tactical, e.g., capacity expansion and reduction. FLP applications are seen in various fields such as supply chain management (Melo et al., 2009; Weskamp et al., 2019), emergency humanitarian relief (Boonmee et al., 2017; Dönmez et al., 2021), warehousing and storage (Coniglio et al., 2017), healthcare (Ahmadi-Javid et al., 2017), distribution systems design (Klose and Drexl, 2005) and road freight transportation (Allen et al., 2012). In the service sector, it is common to describe a facility's maximum capacity in terms of modules. A module consists of, for say, several different rooms (i.e., physical quantification) or several staff members (i.e., personnel quantification) (Correia and Captivo, 2003). This differentiation led to the family of facility location problems with modular capacity.

The decisions made in an FLP are expected to remain functional, for the long-term, in the face of various uncertainties such as demand, transportation costs, facility disruptions, and others. Snyder (2006) provided an early review of facility location under uncertainty. In general, the uncertainty is classified into three groups: supplier-end (e.g., facility disruptions), receiver-end (e.g., varying demand), and in-between (e.g., variations in transportation costs). Two common paradigms to quantify the risk associated with the uncertainty in the objective function are robust optimization (RO) (Ben-Tal et al., 2009) and stochastic optimization (SP) (Birge and Louveaux, 2011). SP typically requires some underlying distributional assumptions about the uncertain parameters to determine the optimal expected objective function value, while robust optimization only assumes that the uncertain parameters belong to a known uncertainty set to optimize the objective function value under the worst-case scenario. Chance constraints are another technique used for problems where uncertainty is present only in the constraints (Gülpınar et al., 2013).

Generally, in optimization under uncertainty, the nature of the decisions and the temporal modeling framework are key elements characterizing the problem's type. Static RO facility location problems assume all decisions to be independent of any future realized uncertainty and are implemented at the outset of the problem (Baron et al., 2011; Zetina et al., 2017). Adaptive RO/SP facility location problems introduce static (uncertainty-independent) decisions at the strategic level, and recourse (uncertainty-dependent) decisions at the tactical level. The two-stage setting assumes the uncertainty is revealed all in one event and the recourse decisions are implemented once. Atamtürk and Zhang (2007) implemented a two-stage RO network flow problem under uncertain demand. Zeng and Zhao (2013) developed a column and constraint generation algorithm for a two-stage robust optimization problem using a reduced vertex enumeration scheme. Ardestani-Jaafari and Delage (2018) presented a location-transportation problem under demand uncertainty as a two-stage robust optimization problem and approximated the adaptive decisions using affine approximations. Siddiq (2013) formulated a robust facility location problem under demand uncertainty, in the healthcare sector, as a two-stage robust optimization problem. Cheng et al. (2021) implemented a two-stage robust facility location problem with demand and facility disruption uncertainty and used a column-and-constraint generation solution technique. Noyan et al. (2016) introduced a two-stage stochastic problem for a humanitarian relief network considering demand and network-related uncertainties and used a branch-and-cut solution algorithm based on Benders decomposition. Albareda-Sambola et al. (2011) presented a two-stage stochastic facility location problem with Bernoulli demand where they considered two different recourse functions and provided a close-form solution for each. In the case where the uncertainty is gradually revealed over time, a multistage-setting is considered (Correia et al., 2018). A multistage framework provides the opportunity to introduce capacity expansion or reduction decisions (Jena et al., 2015) and facility opening or closing decisions (Jena et al., 2016), which offer the ability to design a flexible network for long-term planning. Correia and Melo (2021) introduced a stochastic multiperiod capacitated facility location problem with valid inequalities. The model includes adaptive binary decisions to add capacity to facilities in future periods. The computational cost of the model is high and the uncertainty was approximated with just five discrete scenarios.

Two common solution algorithms for stochastic optimization are scenario-based and decision rule-based methods. In the former, a discrete set of scenarios representing the uncertainty is used to compute the expected objective function value and approximate the optimization problem with a deterministic counterpart. Scenarios are often represented via a scenario tree whose size increases exponentially, in multistage settings, with the size of the decision-making sequence. Decision rule-based methods were first introduced by Garstka and Wets (1974). The prominence of the approach was not fully appreciated until Ben-Tal et al. (2004) illustrated that the linear decision rule (LDR) approximation for robust optimization can be solved in polynomial time. A similar conclusion can be derived for stochastic optimization. The solution approach couples the ideas of approximating recourse decisions as functions or rules of the uncertain parameters with the strong duality theorem, for convex optimization problems, to construct an equivalent deterministic counterpart model. Although the latter poses limitations on the combination of decisions rules and uncertainty set types that satisfy the strong duality theorem, the LDR-based approach scales better than the scenario-based approach.

1.2. Contributions and paper organization

The contributions of this paper are threefold:

1. We formulate a two-stage stochastic facility location problem with modular capacity under uncertain demand for designing the Hydro-Québec logistics network. We revisit the scenario- and linear decision rule-based solution methods for the problem. We conduct computational experiments to explore the

quality of the solutions produced by both methods. The location decisions are then evaluated in a simulator. On the one hand, the scenario-based approach harnesses the distributional information to generate less conservative location decisions; nonetheless, the exponential increase in computational time for the higher dimensional problems is evident. On the other hand, the computational prominence of LDR-based methods comes at the cost of probable overly robust location decisions.

2. Observing that the robustness of the LDR-generated location decisions is not favorable for long-term planning, we propose two heuristics to tune the LDR robustness based on the decision-maker's attitude to risk. We use the value at risk measure and the facility utilization levels (i.e., percentage of maximum capacity utilization), as a feedback mechanism for the obtained location decisions quality. In brief, a decision-maker chooses, at the outset of the planning period, a facility at location i with a maximum capacity level k. If the simulator's output shows that at the 99% value at risk cost scenario, the maximum capacity level $k' < k$ suffices, we then truncate the model-based uncertainty set to eliminate the surplus capacity defined as the difference between k and k'. It is worth noting that the simulator is always executed with the real, non-truncated, uncertainty set.

3. We apply the LDR robust trade-off tuning heuristics to an industrial case study with 14 sites and 474 customers. Two types of uncertain demand distributions are investigated: uniform and rightly-skewed beta distributions. We empirically illustrate the benefits obtained under both demand distributions, and we highlight the impact of the uncertainty distribution on the planning decision.

2. Problem definition and mathematical models

In this section, we first define the problem and introduce a deterministic model, which will serve as a foundation for the stochastic model. We then present the two-stage stochastic formulation and its elements.

2.1. Problem definition

Hydro-Québec has to meet the demand of its customers at minimum cost. To this end, it has to locate its facilities (i.e., service centers), select their capacity levels, decide on the actual number of modules (i.e., service teams) in each facility and determine the optimal delivery of services between facilities and customers. We define the set of sites where a new facility may be located as $\mathcal{I} = \{1, 2, ..., |\mathcal{I}|\}$. The capacity levels, in terms of number of modules, for a facility in site i are given by $\mathcal{K}_i = \{1, 2, ..., |\mathcal{K}_i|\}$. Each facility hosts service teams offering to deliver a set of services or products given by $\mathcal{P} = \{1, 2, ..., |\mathcal{P}|\}$. The set of customers is represented by $\mathcal{J} = \{1, 2, ..., |\mathcal{J}|\}$.

2.2. Deterministic model

A deterministic model representing the facility location problem is given in model (1). The binary location variable y_{ik} is equal to 1 if a facility of capacity level k is opened at site i, and 0 otherwise. The number of hours of service of type p (i.e., flow) provided by service teams in site i to customer j is denoted by the continuous variable x_{ijp}. The discrete number of modules (i.e., service teams) providing a service p in site i is given by the discrete decision n_{ip}. The objective function (1a) minimizes the total costs related to locating a facility of capacity level k in site i (i.e., F_{ik}), the transportation costs corresponding to delivering service p by a service team in site i to customer j (i.e., C_{ijp}) and the operating costs of service teams providing service p in site i (i.e., H_{ip}).

Constraints (1b) dictate that the customer demands for all products must be satisfied. The demand of customer j for service p is given by D_{jp}. The facility supply constraints are given in (1c) and the facility module capacity constraints are given in (1d). Here, Q_p is equal to the total service hours of type p provided by a single module and N_{ik} is the maximum number of modules that can be installed at site i when capacity level k is chosen. The parameter I_{ip} is the initial number of modules providing service p in site i. A positive I_{ip} allows the inclusion of existing facilities in the model, with no option of reducing the initial number of modules or closing the facility. Only a single facility is allowed in site i and this requirement is guaranteed by constraints (1e). Lastly, constraints (1f)−(1h) specify the domains of the variables.

$$
\begin{aligned}
\min \quad & \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} F_{ik}y_{ik} + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}} C_{ijp}x_{ijp} + \sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} H_{ip}n_{ip} && && \text{(1a)}\\
\text{s.t.} \quad & \sum_{i\in\mathcal{I}} x_{ijp} \geq D_{jp} && j\in\mathcal{J},\ p\in\mathcal{P} && \text{(1b)}\\
& \sum_{j\in\mathcal{J}} x_{ijp} \leq Q_p\Big(I_{ip}+n_{ip}\Big) && i\in\mathcal{I},\ p\in\mathcal{P} && \text{(1c)}\\
& \sum_{p\in\mathcal{P}} \Big(I_{ip}+n_{ip}\Big) \leq \sum_{k\in\mathcal{K}_i} N_{ik}y_{ik} && i\in\mathcal{I} && \text{(1d)}\\
& \sum_{k\in\mathcal{K}_i} y_{ik} \leq 1 && i\in\mathcal{I} && \text{(1e)}\\
& y_{ik}\in\{0,1\} && i\in\mathcal{I},\ k\in\mathcal{K}_i && \text{(1f)}\\
& x_{ijp}\geq 0 && i\in\mathcal{I},\ p\in\mathcal{P},\ j\in\mathcal{J} && \text{(1g)}\\
& n_{ip}\geq 0 \text{ and integer} && i\in\mathcal{I},\ p\in\mathcal{P} && \text{(1h)}
\end{aligned}
$$

2.3. Two-stage stochastic optimization model

To address the uncertainty in the customer demands D_{jp} for all j and p, we introduce a primitive uncertainty vector $\boldsymbol{\xi} = \{\xi_{11},\ldots,\xi_{1|\mathcal{P}|},\ldots,\xi_{|\mathcal{J}|1},\ldots,\xi_{|\mathcal{J}||\mathcal{P}|}\} \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{P}|}$ where $D_{jp}(\boldsymbol{\xi}) = \mathbf{e}_{jp}^{\top}\boldsymbol{\xi} = \xi_{jp}$. We assume the uncertain customer demands belong to an interval-based uncertainty set Ξ, i.e., a hyper-rectangle, which is defined as a box in equation (2):

$$
\Xi := \left\{\boldsymbol{\xi}\in\mathbb{R}^{|\mathcal{J}|\times|\mathcal{P}|}\ \middle|\ \mathbf{W}\boldsymbol{\xi}\geq\mathbf{h},\ \mathbf{W}\in\mathbb{R}^{m\times(|\mathcal{J}|\times|\mathcal{P}|)},\ \mathbf{h}\in\mathbb{R}^m\right\}. \tag{2}
$$

Though a multi-stage framework is preferred for long term planning to characterize the uncertain demand, we consider a two-stage stochastic optimization framework where the uncertain demands are revealed once and for all. The latter framework provides preliminary high-level decisions insights on the Hydro-Québec network design, at low computational costs. The location decisions y_{ik} for all i and k are taken at the outset of the problem and set to be static. The flows $x_{ijp}(\boldsymbol{\xi})$ and the number of modules $n_{ip}(\boldsymbol{\xi})$ are adaptive, and these decisions are taken after the realization of the customer demands. The two-stage stochastic formulation is presented in model (3) where the location costs and expected (i.e., $\mathbb{E}[.]$) transportation and modules costs are minimized. The number of modules is relaxed to be continuous. In our application, the relaxation does not have a significant impact since a fractional number of modules means the existence of either service teams having over-time shifts or an additional single service team with a part-time shift.

$$\min \quad \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} F_{ik}y_{ik} + \mathbb{E}\left[\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}} C_{ijp}x_{ijp}(\boldsymbol{\xi}) + \sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} H_{ip}n_{ip}(\boldsymbol{\xi})\right] \tag{3a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{I}} x_{ijp}(\boldsymbol{\xi}) \geq D_{jp}(\boldsymbol{\xi}) \qquad j\in\mathcal{J},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi \tag{3b}$$

$$\sum_{j\in\mathcal{J}} x_{ijp}(\boldsymbol{\xi}) \leq Q_p\Big(I_{ip} + n_{ip}(\boldsymbol{\xi})\Big) \qquad i\in\mathcal{I},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi \tag{3c}$$

$$\sum_{p\in\mathcal{P}} \Big(I_{ip} + n_{ip}(\boldsymbol{\xi})\Big) \leq \sum_{k\in\mathcal{K}_i} N_{ik}y_{ik} \qquad i\in\mathcal{I},\ \boldsymbol{\xi}\in\Xi \tag{3d}$$

$$\sum_{k\in\mathcal{K}_i} y_{ik} \leq 1 \qquad i\in\mathcal{I} \tag{3e}$$

$$y_{ik}\in\{0,1\} \qquad i\in\mathcal{I},\ k\in\mathcal{K}_i \tag{3f}$$

$$x_{ijp}(\boldsymbol{\xi}) \geq 0 \qquad i\in\mathcal{I},\ p\in\mathcal{P},\ j\in\mathcal{J},\ \boldsymbol{\xi}\in\Xi \tag{3g}$$

$$n_{ip}(\boldsymbol{\xi}) \geq 0 \qquad i\in\mathcal{I},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi \tag{3h}$$

3. Solution approximation methods

The general form of model (3) is computationally intractable due to the presence of semi-infinite constraints which have to be satisfied over an uncertainty set (i.e., infinite number of constraints). Scenario- and LDR-based solution methods circumvent the intractability through different approximation schemes. On the one hand, the scenario-based method approximates the uncertainty set via a set of discrete scenarios. It then reformulates the adaptive decisions and the semi-infinite constraints as a set of deterministic decisions and constraints at each discrete scenario, respectively. On the other hand, the LDR-based method assumes an a-priori function or rule to approximate the adaptive decisions and it exploits the strong duality theorem, under closed convex uncertainty set, to reformulate the tractable deterministic counterpart. Further descriptions of both methods are introduced subsequently.

3.1. Scenario-based solution method

Two questions arise when implementing the scenario-based approximation method: *"How to sample the discrete scenarios?"* and *"What is the scenario set size?"*. For sampled scenarios, sampling is done by either assuming a specific distribution or, ideally, inferring distributional information from historical data. As for the scenario set size, generating a trivial "large" set of scenarios offers a more refined solution approximation. It asymptotically converges to the true solution (Kleywegt et al., 2002). Nonetheless, it is computationally prohibitive for large-scale problems as the model and time complexity increase exponentially.

For a scenario set $\mathcal{S}$, the expectation in the objective function (3a) is approximated using an average measure as in equation (4a) where x_{ijps} and n_{ips} are deterministic decisions for each scenario s representing the flow and number of modules, respectively. The uncertain demand is approximated by a set of discrete demand values D_{jps} for all $s\in\mathcal{S}$ and each semi-infinite constraint is reformulated as a set of deterministic constraints for each $s\in\mathcal{S}$. The scenario-based approximated stochastic optimization problem is given in model (4):

$$
\begin{aligned}
\min \quad & \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} F_{ik}y_{ik} + \frac{1}{|S|}\sum_{s\in\mathcal{S}}\left[\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}} C_{ijp}x_{ijps} + \sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} H_{ip}n_{ips}\right] && && \text{(4a)}\\
\text{s.t.} \quad & \sum_{i\in\mathcal{I}} x_{ijps} \geq D_{jps} && j\in\mathcal{J},\ p\in\mathcal{P},\ s\in\mathcal{S} && \text{(4b)}\\
& \sum_{j\in\mathcal{J}} x_{ijps} \leq Q_p\Big(I_{ip} + n_{ips}\Big) && i\in\mathcal{I},\ p\in\mathcal{P},\ s\in\mathcal{S} && \text{(4c)}\\
& \sum_{p\in\mathcal{P}} \Big(I_{ip} + n_{ips}\Big) \leq \sum_{k\in\mathcal{K}_i} N_{ik}y_{ik} && i\in\mathcal{I},\ s\in\mathcal{S} && \text{(4d)}\\
& \sum_{k\in\mathcal{K}_i} y_{ik} \leq 1 && i\in\mathcal{I} && \text{(4e)}\\
& y_{ik} \in \{0,1\} && i\in\mathcal{I},\ k\in\mathcal{K}_i && \text{(4f)}\\
& x_{ijps} \geq 0 && i\in\mathcal{I},\ p\in\mathcal{P},\ j\in\mathcal{J}, s\in\mathcal{S} && \text{(4g)}\\
& n_{ips} \geq 0 && i\in\mathcal{I},\ p\in\mathcal{P},\ s\in\mathcal{S} && \text{(4h)}
\end{aligned}
$$

3.2. Linear decision rule-based solution method

The linear decision rule-based solution method is an alternative technique to circumvent the intractability of model (3). The main concept is based on assuming a linear dependence, or rule, of the adaptive decisions on the uncertain parameters, then exploiting the strong duality theorem, under a box uncertainty set, to construct an equivalent linear deterministic counterpart.

3.2.1. Constructing the equivalent tractable counterpart

The flow $x_{ijp}(\boldsymbol{\xi})$ and number of modules $n_{ip}(\boldsymbol{\xi})$ policies in model (3) are approximated via linear decision rules in equation (5), where $x^0_{ijp},\ n^0_{ip} \in \mathbb{R}$ are the intercepts and $\mathbf{X}^1_{ijp},\ \mathbf{N}^1_{ip} \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{P}|}$ are the slopes. The intercepts and the slopes are both decision variables to be optimized.

$$
\begin{aligned}
x_{ijp}(\boldsymbol{\xi}) &= x^0_{ijp} + (\mathbf{X}^1_{ijp})^\top\boldsymbol{\xi} && i\in\mathcal{I},\ j\in\mathcal{J},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi && \text{(5a)}\\
n_{ip}(\boldsymbol{\xi}) &= n^0_{ip} + (\mathbf{N}^1_{ip})^\top\boldsymbol{\xi} && i\in\mathcal{I},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi && \text{(5b)}
\end{aligned}
$$

Replacing the LDRs in model (3), we get

$$
\begin{aligned}
\min \quad & \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}_i} F_{ik}y_{ik} + \mathbb{E}\left[\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}} C_{ijp}\Big[x^0_{ijp} + (\mathbf{X}^1_{ijp})^\top\boldsymbol{\xi}\Big] + \sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} H_{ip}\Big[n^0_{ip} + (\mathbf{N}^1_{ip})^\top\boldsymbol{\xi}\Big]\right] && && \text{(6a)}\\
\text{s.t.} \quad & \sum_{i\in\mathcal{I}} \big(x^0_{ijp} + (\mathbf{X}^1_{ijp})^\top\boldsymbol{\xi}\big) \geq \mathbf{e}^\top_{jp}\boldsymbol{\xi} && j\in\mathcal{J},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi && \text{(6b)}\\
& \sum_{j\in\mathcal{J}} \big(x^0_{ijp} + (\mathbf{X}^1_{ijp})^\top\boldsymbol{\xi}\big) \leq Q_p\Big(I_{ip} + n^0_{ip} + (\mathbf{N}^1_{ip})^\top\boldsymbol{\xi}\Big) && i\in\mathcal{I},\ p\in\mathcal{P},\ \boldsymbol{\xi}\in\Xi && \text{(6c)}\\
& \sum_{p\in\mathcal{P}} \Big(I_{ip} + n^0_{ip} + (\mathbf{N}^1_{ip})^\top\boldsymbol{\xi}\Big) \leq \sum_{k\in\mathcal{K}_i} N_{ik}y_{ik} && i\in\mathcal{I},\ \boldsymbol{\xi}\in\Xi && \text{(6d)}\\
& \sum_{k\in\mathcal{K}_i} y_{ik} \leq 1 && i\in\mathcal{I} && \text{(6e)}
\end{aligned}
$$

$$y_{ik} \in \{0,1\} \qquad i \in \mathcal{I},\ k \in \mathcal{K}_i \tag{6f}$$

$$x^0_{ijp} + (\mathbf{X}^1_{ijp})^\top \boldsymbol{\xi} \geq 0 \qquad i \in \mathcal{I},\ p \in \mathcal{P},\ j \in \mathcal{J},\ \boldsymbol{\xi} \in \Xi \tag{6g}$$

$$n^0_{ip} + (\mathbf{N}^1_{ip})^\top \boldsymbol{\xi} \geq 0 \qquad i \in \mathcal{I},\ p \in \mathcal{P},\ \boldsymbol{\xi} \in \Xi, \tag{6h}$$

where $\mathbf{e}^\top_{jp}\boldsymbol{\xi} = \xi_{jp}$. As a way of illustrating the technique that derives the finite tractable counterpart of model (6), consider the constraints

$$\sum_{i \in \mathcal{I}} \left(x^0_{ijp} + (\mathbf{X}^1_{ijp})^\top \boldsymbol{\xi}\right) \geq \mathbf{e}^\top_{jp}\boldsymbol{\xi}, \qquad j \in \mathcal{J},\ p \in \mathcal{P},\ \boldsymbol{\xi} \in \Xi, \tag{7}$$

which are equivalent to

$$\sum_{i \in \mathcal{I}} x^0_{ijp} + \left\{ \begin{array}{ll} \min\limits_{\boldsymbol{\xi} \in \Xi} & \left(\sum\limits_{i \in \mathcal{I}} \mathbf{X}^1_{ijp} - \mathbf{e}_{jp}\right)^\top \boldsymbol{\xi} \\ \text{s.t.} & \mathbf{W}\boldsymbol{\xi} \geq \mathbf{h} \end{array} \right\} \geq 0, \qquad j \in \mathcal{J},\ p \in \mathcal{P}. \tag{8}$$

Stated differently, the solution obtained from (8) satisfies constraints (7) for any realization $\boldsymbol{\xi} \in \Xi$. The dual of the inner minimization problem is given in (9), where $\boldsymbol{\lambda}_{jp}$ are the dual variables:

$$\sum_{i \in \mathcal{I}} x^0_{ijp} + \left\{ \begin{array}{ll} \max\limits_{\boldsymbol{\lambda}_{jp}} & \mathbf{h}^\top \boldsymbol{\lambda}_{jp} \\ \text{s.t.} & \mathbf{W}^\top \boldsymbol{\lambda}_{jp} = \sum\limits_{i \in \mathcal{I}} \mathbf{X}^1_{ijp} - \mathbf{e}_{jp} \\ & \boldsymbol{\lambda}_{jp} \in \mathbb{R}^m_+ \end{array} \right\} \geq 0, \qquad j \in \mathcal{J},\ p \in \mathcal{P} \tag{9}$$

Equations (8) and (9) are equivalent since the inner optimization problems have the same optimal solution based on linear programming strong duality property. The max operator is dropped from the final formulation of the finite counterpart in equation (10) without affecting feasibility (i.e., using the same logic when introducing the *min* operator earlier in (8)):

$$\sum_{i \in \mathcal{I}} x^0_{ijp} + \mathbf{h}^\top \boldsymbol{\lambda}_{jp} \geq 0 \qquad j \in \mathcal{J},\ p \in \mathcal{P} \tag{10a}$$

$$\mathbf{W}^\top \boldsymbol{\lambda}_{jp} = \sum_{i \in \mathcal{I}} \mathbf{X}^1_{ijp} - \mathbf{e}_{jp} \qquad j \in \mathcal{J},\ p \in \mathcal{P} \tag{10b}$$

$$\boldsymbol{\lambda}_{jp} \in \mathbb{R}^m_+ \qquad j \in \mathcal{J},\ p \in \mathcal{P} \tag{10c}$$

The same procedure is performed to the remaining semi-infinite constraints to derive the overall stochastic linear finite counterpart in model (11):

$$\min \quad \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} F_{ik} y_{ik} + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}} C_{ijp}\Big(x^0_{ijp} + \big(\mathbf{X}^1_{ijp}\big)^\top \mathbb{E}[\boldsymbol{\xi}]\Big) + \sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} H_{ip}\Big(n^0_{ip} + \big(\mathbf{N}^1_{ip}\big)^\top \mathbb{E}[\boldsymbol{\xi}]\Big) \tag{11a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{I}} x^0_{ijp} + \mathbf{h}^\top \boldsymbol{\lambda}_{jp} \geq 0 \qquad j \in \mathcal{J},\ p \in \mathcal{P} \tag{11b}$$

$$\mathbf{W}^\top \boldsymbol{\lambda}_{jp} = \sum_{i \in \mathcal{I}} \mathbf{X}^1_{ijp} - \mathbf{e}_{jp} \qquad j \in \mathcal{J},\ p \in \mathcal{P} \tag{11c}$$

$$Q_p\Big(I_{ip}+n^0_{ip}\Big)-\sum_{j\in\mathcal{J}}x^0_{ijp}+\mathbf{h}^\top\boldsymbol{\mu}_{ip}\ge 0 \qquad i\in\mathcal{I},\ p\in\mathcal{P} \tag{11d}$$

$$\mathbf{W}^\top\boldsymbol{\mu}_{ip}=Q_p\mathbf{N}^1_{ip}-\sum_{j\in\mathcal{J}}\mathbf{X}^1_{ijp} \qquad i\in\mathcal{I},\ p\in\mathcal{P} \tag{11e}$$

$$\sum_{k\in\mathcal{K}_i}N_{ik}y_{ik}-\sum_{p\in\mathcal{P}}\Big(I_{ip}+n^0_{ip}\Big)+\mathbf{h}^\top\boldsymbol{\tau}_i\ge 0 \qquad i\in\mathcal{I} \tag{11f}$$

$$\mathbf{W}^\top\boldsymbol{\tau}_i=-\sum_{p\in\mathcal{P}}\mathbf{N}^1_{ip} \qquad i\in\mathcal{I} \tag{11g}$$

$$x^0_{ijp}+\mathbf{h}^\top\boldsymbol{\gamma}_{ijp}\ge 0 \qquad i\in\mathcal{I},\ j\in\mathcal{J},\ p\in\mathcal{P} \tag{11h}$$

$$\mathbf{W}^\top\boldsymbol{\gamma}_{ijp}=\mathbf{X}^1_{ijp} \qquad i\in\mathcal{I},\ j\in\mathcal{J},\ p\in\mathcal{P} \tag{11i}$$

$$n^0_{i,p}+\mathbf{h}^\top\boldsymbol{\zeta}_{ip}\ge 0 \qquad i\in\mathcal{I},\ p\in\mathcal{P} \tag{11j}$$

$$\mathbf{W}^\top\boldsymbol{\zeta}_{ip}=\mathbf{N}^1_{ip} \qquad i\in\mathcal{I},\ p\in\mathcal{P} \tag{11k}$$

$$\sum_{k\in\mathcal{K}_i}y_{ik}\le 1 \qquad i\in\mathcal{I} \tag{11l}$$

$$y_{ik}\in\{0,1\} \qquad i\in\mathcal{I},\ k\in\mathcal{K}_i \tag{11m}$$

$$\boldsymbol{\lambda}_{jp}\in\mathbb{R}^m_+ \qquad j\in\mathcal{J},\ p\in\mathcal{P} \tag{11n}$$

$$\boldsymbol{\gamma}_{ijp}\in\mathbb{R}^m_+ \qquad i\in\mathcal{I},\ j\in\mathcal{J},\ p\in\mathcal{P} \tag{11o}$$

$$\boldsymbol{\mu}_{ip},\boldsymbol{\zeta}_{ip}\in\mathbb{R}^m_+ \qquad i\in\mathcal{I},\ p\in\mathcal{P} \tag{11p}$$

$$\boldsymbol{\tau}_i\in\mathbb{R}^m_+ \qquad i\in\mathcal{I} \tag{11q}$$

3.2.2. LDR solution quality evaluation

The decision rule-based solution method constructs an equivalent tractable counterpart for a two-stage stochastic optimization problem without a reference to the distributional nature of the uncertainty, other than in the objective function. This is seen in equation (8), where the only required uncertainty information is the box representation. The LDR approximation provides a competitive edge in terms of computational complexity in comparison to the scenario-based approximation. However, it raises the following question: What is the "true" first-stage location decisions quality under the uncertainty distribution?

To answer this question, we evaluate the optimal first-stage decisions of an LDR-based method (i.e., a form of look-ahead model policies) in a simulator (Powell, 2014). That is, we implement the optimal decision at the current time stage t (i.e., $t=1$), we observe the realized demand uncertainty based on a given distribution, we roll the horizon to the next time stage $t+1$ (i.e., $t=2$) and re-optimize the problem given the new demand information. In a two-stage setting, we have only one additional deterministic optimization per sampled scenario s at $t=2$. In our FLP, the optimal LDR location decisions are evaluated using the set $\mathcal{S}'$. The simulator is executed for all scenarios in $\mathcal{S}'$ and the expected second-stage cost is computed as the average of the simulated second-stage costs over $s\in\mathcal{S}'$. Based on the LDR method nature, all demand scenarios in $s\in\mathcal{S}'$ are feasible. For comparison purposes, the scenario-based first-stage decisions are evaluated in the same simulator using the same scenario set, used for the LDR solution, where $|S'|>>|S|$. In this case, it is possible to have infeasible scenarios (i.e., demand scenarios that could not be satisfied by the located facilities). We compute the average second-stage cost based on the feasible scenarios and report the number of infeasible scenarios.

4. Robustness trade-off tuning heuristic

The facility location problem we consider in this paper dictates the fulfillment of all customer demands as in equation (1b). The LDR solution guarantees the requirements over the entire uncertainty set. Nonetheless, the approach may lead to overly conservative first-stage location decisions. For example, assume there are three customers, each with an uncertain demand following a uniform distribution between 0 and 100. The probability that a customer's demand falls between 80 and 100 is 0.2. Knowing that the demand distributions are independent, the probability that all three demand values fall between from 80 to 100 is $0.2^3 = 0.016$ (i.e., 1.6%). If there are 10 customers with the same distributional information, the probability that all uncertain demand values fall within the range from 80 to 100 is $1.024e^{-7}$, which is equivalent to 1 in 10 million demand scenarios. In practice, the latter event can be considered as negligible. However, the LDR model-based solution does anticipate for this improbable event. Consequently, it may output overly robust location decisions.

To fine tune the robustness and conservative nature of the LDR location decisions, we propose two tuning heuristic algorithms. The first is more vigorous towards risk, defined here by the number of infeasible or unfulfilled demand scenarios, and it is more systematic. While the second is greedier and more efficient in reducing the location decisions' robustness, it comes with higher risk. Both tuning heuristic algorithms are based on two key elements: the value at risk measure $\mathrm{VaR}_\alpha(.)$ and the facility utilization level in site i of capacity level k, l_{ik}, at the α% value at risk simulated cost scenario. Value at risk is a measure first introduced in the financial sector to measure the investment risk loss or returns with a given confidence value α. For example, an investment or portfolio with a VaR_{95} risk loss of 1 million dollars is equivalent to saying that the actual loss, with a 95% probability, is less than or equal to 1 million dollars. We use $\mathrm{VaR}_\alpha(.)$ as a post-hoc measure to reduce the robustness of an LDR location decisions, while ensuring that the capacity is respected at a high probability in the simulator.

In our cost minimization facility location problem, the cost is directly correlated with the demand values. The higher the demand values, the higher the cost. The cost distribution is approximated when evaluating the LDR location decisions in a simulator. Facility utilization rate l_{ik} at the α% value at risk simulated cost scenario is equal to the fraction or percentage of the maximum facility capacity (e.g., in terms of service hours) at the aforementioned scenario. For example, for a given facility with 15 modules at 99% value at risk simulated cost scenario and a 20 modules maximum capacity, the utilization level is equal to 0.75 or 75% for 99% of the time.

The LDR Robustness Tuning Heuristic 1 is shown in Algorithm 1. After simulating the location decisions in a simulator, the total modules (i.e., service teams) surplus capacity $\hat{N}$ is computed facility-wise via the l_{ik} at the α% value at risk simulated cost scenario. For each located facility in site i and capacity level k, if $l_{ik}N_{ik} \leq N_{ik'}$ where $k' < k$ and $k,\ k' \in \mathcal{K}_i$, then for 99% of the time, a facility with a module capacity level k' in site i is sufficient. The difference between N_{ik} and $N_{ik'}$ is denoted as the facility module surplus. The same condition is applied to all facilities to compute $\hat{N}$. Then, the heuristic feeds this information back to the model via an updated definition of the model-based uncertainty set (i.e., Δ^t). The truncated uncertainty set is guaranteed to generate the same or better set of location decisions in the next iteration. We emphasize that the LDR solution obtained using the updated model-based uncertainty set is evaluated in the simulator using the real, non-truncated, uncertainty set. Model-based LDR location decisions may be deemed infeasible for some demand scenarios in $\mathcal{S}'$. We introduce the maximum allowed infeasible scenarios, ϵ, as a percentage of $\mathcal{S}'$. The heuristic terminates when the total module surplus capacity is zero or the maximum infeasible simulated scenario percentage exceeds ϵ.

Algorithm 1 LDR Robustness Tuning Heuristic 1

Require: $\bar{\xi}_{jp},\ \Delta,\ \mathcal{S}',\ \epsilon$

  Initialize $t = 1$ and $\Delta^t = \Delta$

  repeat

    $\Xi \leftarrow \xi_{jp} \in \left[(1-\Delta^t)\bar{\xi}_{jp},\ (1+\Delta^t)\bar{\xi}_{jp}\right]\ \forall j, p$

    Solve (10) to obtain y^*_{ik} for all i, k

    Simulate for n_{ips} and x_{ijps} using y^*_{ik} and $\mathcal{S}'$

    $\hat{\epsilon} \leftarrow$ Percentage of infeasible scenarios within simulator

    For $y^*_{ik} = 1$, compute l_{ik} at 99% VaR cost scenario.

    $\hat{N} = 0$

    $N^{\max} = 0$

    for each l_{ik} **do**

      $N^{\max} = N^{\max} + N_{ik}$

      for $k' = 1 : k-1$ **do**

        if $l_{ik}N_{ik} \leq N_{ik'}$ **then**

          $\hat{N} = \hat{N} + (N_{ik} - N_{ik'})$

          Break from the k' for-loop

        end if

      end for

    end for

    if $\hat{N} > 0$ **then**

      $\hat{Q} = \hat{N} \min_{p\in\mathcal{P}} Q_p$

      $Q^{\max} = N^{\max} \min_{p\in\mathcal{P}} Q_p$

      $t = t + 1$ ▷ Iteration index for new/truncated model-based demand variation

      if $(Q^{\max} - \hat{Q}) < \sum_j \sum_p (1+\Delta^t)\bar{\xi}_{jp}$ **then**

        $\Delta^t = \dfrac{Q^{\max} - \hat{Q}}{\sum_j \sum_p \bar{\xi}_{jp}} - 1$

      else

        $\Delta^t = \dfrac{\sum_j \sum_p (1+\Delta^t)\bar{\xi}_{jp} - \hat{Q}}{\sum_j \sum_p \bar{\xi}_{jp}} - 1$

      end if

    end if

  until $\hat{N} = 0$ or $\hat{\epsilon} > \epsilon$

The LDR Robustness Tuning Heuristic 2 is shown in Algorithm 2. The main difference is in the way a facility module (i.e., service team) surplus is defined. For each located facility in site i of capacity level k, the surplus is defined as $(1 - l_{ik})N_{ik}$ regardless of the value of the facility utilization level l_{ik} at the 99% value at risk simulated cost scenario. The heuristic terminates when there is no change in the new model-based uncertainty (i.e., no change in the nominal value deviation Δ^t) or when the percentage of infeasible scenarios within the simulator exceeds the maximum allowed.

Algorithm 2 LDR Robustness Tuning Heuristic 2

Require: $\bar{\xi}_{jp},\ \Delta,\ \mathcal{S}',\ \epsilon$

Initialize $t = 1$ and $\Delta^t \equiv \Delta^1 = \Delta$

repeat

$\Xi \leftarrow \xi_{jp} \in \Big[(1-\Delta^t)\bar{\xi}_{jp},\ (1+\Delta^t)\bar{\xi}_{jp}\Big]\ \forall j,p$

Solve equation (10) $\rightarrow y_{ik}$ for all i, k

Simulate for n_{ips} and x_{ijps} using y_{ik} and $\mathcal{S}'$

$\hat{\epsilon} \leftarrow$ Percentage of infeasible scenarios in S'

For $y^*_{ik} = 1$, compute l_{ik} at 99% VaR cost scenario.

$\hat{N} = 0$

$N^{\max} = 0$

for each l_{ik} **do**

$\hat{N} = \hat{N} + \big(1 - l_{ik}\big)N_{ik}$

$N^{\max} = N^{\max} + N_{ik}$

end for

$\hat{Q} = \hat{N}\min_{p\in\mathcal{P}} Q_p$

$Q^{\max} = N^{\max}\min_{p\in\mathcal{P}} Q_p$

$t = t + 1$ ▷ Iteration index for new/truncated model-based demand variation

if $(Q^{\max} - \hat{Q}) < \sum_j \sum_p (1+\Delta^t)\bar{\xi}_{jp}$ **then**

$$\Delta^t = \frac{Q^{\max} - \hat{Q}}{\sum_j \sum_p \bar{\xi}_{jp}} - 1$$

else

$$\Delta^t = \frac{\sum_j \sum_p (1+\Delta^t)\bar{\xi}_{jp} - \hat{Q}}{\sum_j \sum_p \bar{\xi}_{jp}} - 1$$

end if

until $\Delta^t = \Delta^{t-1}$ or $\hat{\epsilon} > \epsilon$

5. Computational results

In this section, we first conduct an illustrative numerical experiment to highlight LDR scaling features and demonstrate the two LDR robustness tuning heuristics for the overly robust solutions. We then introduce the industrial instance and discuss the benefits and decision insights obtained from implementing the proposed solution technique.

5.1. Illustrative numerical experiment

For the computational setting, we consider a super set for both facilities $\mathcal{I}$ and customers $\mathcal{J}$ equal to $\{5, 10, 15, 20\}$. We assume a single product, $|\mathcal{P}| = 1$, and five possible facility maximum capacity levels for each site i (i.e., $\mathcal{K}_i = \{20, 40, 60, 80, 100\}$ and $|\mathcal{K}_i| = 5$). In the scenario-based approach, the number of scenarios in $\mathcal{S}$ is 3000, which is high enough to generate a solution with less than 1% optimality gap according to our preliminary analysis. The uncertain demands of the customers are assumed to have nominal values randomly sampled from the set $\{300, 400, 500\}$ and with $\pm 20\%$ nominal deviations (i.e., $\Delta = 0.2$). For each instance (i.e., $|\mathcal{I}|, |\mathcal{J}|, |\mathcal{P}|, |\mathcal{K}_i| = 5, 5, 1, 5$), both solution approaches have the same cost and uncertainty-related parameters setting. The scenario set $\mathcal{S}'$ used to simulate the location decisions for both methods is also the same. The number of scenarios in $\mathcal{S}'$ is equal to 100,000. For the scenario-based approach, the scenario set used in the model-based solution is not a subset of the scenario set used to simulate the location decisions, $\mathcal{S} \not\subset \mathcal{S}'$. We investigate two demand distributions, in two separate experiments, with different skewness: a uniform and a right-skewed beta ($\alpha = 8,\ \beta = 2$) distribution (e.g., Figure 1). We assume that all customer demands follow the same distribution and that they are independent.

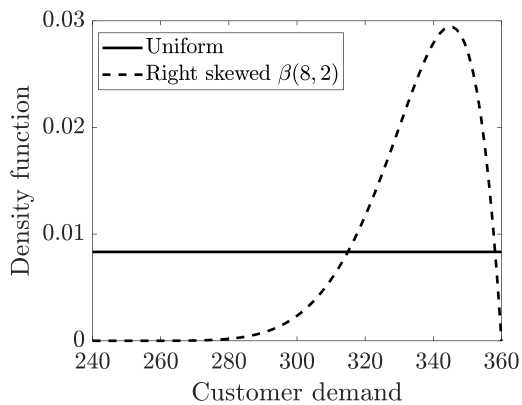


Figure 1: A uniform and a beta distributed customer demand with a nominal value of 300 and $\Delta = 20\%$.

The inclusion of the scenario-based solution methods in the numerical experiments is not to purposefully compare it with the LDR-based solution, though a point can be made regarding the significant solution time reduction offered by LDRs. Rather, the scenario-based solution provides a reference to measure the conservative nature of the LDR location decisions as the former incorporates uncertainty distributional information in approximating the feasible region.

Uniform distributed customer demands

Table 1 lists the location cost, simulated total cost, time, and the percentage of simulated infeasible scenarios $\hat{\epsilon}$ for 16 different $|\mathcal{I}|, |\mathcal{J}|, |\mathcal{P}|, |\mathcal{K}_i|$ instances assuming uniform distributed customer demands. The total location cost is a measure of the location decisions' quality. The higher total location cost exhibited by an LDR solution with respect to the scenario-based solution reflects overly robust location decisions by an LDR. The simulated total cost is a measure of the solution quality, whereas the time is an indication of the computational cost required to solve the model. It is apparent through the numerical experiments that the LDR-based method is computationally more efficient that the scenario-based method. An infeasible scenario is a demand scenario that cannot be satisfied by the model-based location decisions within the simulator. The total number of infeasible scenarios is represented via the percentage of scenarios in $\mathcal{S}'$ (i.e., $\hat{\epsilon}$).

Out of the 16 instances, LDR solution quality is in-par with the scenario solution in six instances (e.g., $|\mathcal{I}|, |\mathcal{J}|, |\mathcal{P}|, |\mathcal{K}_i| = 20, 5, 1, 5$). This is accomplished at a fraction (three to four orders of magnitude less) of the computational time needed for the scenario-based method. In the remaining ten instances, the LDR location cost exceeded that of the scenario-based method. The flexibility of having higher capacity led to a better second-stage cost; nonetheless, the solution quality difference is still significant as portrayed by the simulated total cost change $\delta(\%)$. As for the number of infeasible scenarios, which could not have been fulfilled by the location decisions in the simulator, it is, as expected, null for the LDR-based method. Only three out of 16 instances suffer infeasible simulated scenarios for the scenario-based approach; however, the extent is negligible for two of them (i.e., $\hat{\epsilon} < 0.01\%$). For the $|\mathcal{I}|, |\mathcal{J}|, |\mathcal{P}|, |\mathcal{K}_i| = 5, 15, 1, 5$ instance, there are 53 infeasible scenarios out of 100,000 (i.e., $\hat{\epsilon} = 0.053\%$).

Table 1: LDR- and scenario-based results under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Location cost			Simulated total cost			Time (sec)		$\hat{\epsilon}$ (%)	
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	LDR	Scen.	δ(%)	LDR	Scen.	δ(%)	LDR	Scen.	LDR	Scen.
5,5,1,5	2184.6	2184.6	0	5321.1	5321.1	0	0.05	22.1	0	0
10,5,1,5	2028.6	2028.6	0	4321.8	4321.8	0	0.05	212.5	0	0
15,5,1,5	2150.9	2150.9	0	5138.2	5138.2	0	0.13	85.3	0	0
20,5,1,5	3153.9	3153.9	0	6856.1	6856.1	0	0.22	5196.0	0	0
5,10,1,5	3860.2	3860.2	0	8756.2	8756.2	0	0.09	400.0	0	0
10,10,1,5	5511.6	4383.0	20.5	11994.6	10918.1	9.0	0.17	313.3	0	0
15,10,1,5	5369.6	4388.7	18.3	12205.9	11237.7	7.9	0.31	498.0	0	0
20,10,1,5	4200.4	4200.4	0	12023.9	12023.9	0	0.41	1373.2	0	0
5,15,1,5	8966.2	7079.0	21.0	18722.0	16918.3	9.6	0.28	173.5	0	0.05
10,15,1,5	8136.0	7120.8	12.5	18026.4	17080.4	5.2	0.44	243.0	0	0
15,15,1,5	6452.5	6238.4	3.3	19115.1	18901.0	1.1	0.45	1339.6	0	0
20,15,1,5	6416.3	6201.1	3.4	15493.6	15278.4	1.4	0.70	4869.8	0	0
5,20,1,5	11893.4	9767.2	17.9	25530.0	23636.6	7.4	0.34	86.0	0	0
10,20,1,5	10459.9	9028.1	13.7	28179.9	26759.3	5.0	0.58	1263.5	0	0
15,20,1,5	9640.9	8458.3	12.3	21476.7	20319.9	5.3	5.39	10802.9	0	0
20,20,1,5	10097.9	8469.0	16.1	27346.9	25900.7	5.3	7.45	6593.4	0	0

The optimal locations of the selected facilities and their capacity levels for the 16 instances are shown in Table 2. In addition, for each instance, the facility utilization level at the scenario corresponding to the 99% VaR simulated total cost is computed. The Robustness Tuning Heuristic 1 uses the latter information to identify instances where LDR exhibits overly conservative location decisions. For example, in the $(10, 10, 1, 5)$ instance, the facility utilization level in site 7, of maximum capacity level 5, is 75.49%. The maximum capacities in levels 4 and 5 are 80 and 100, respectively. This entails that for 99% of the time, it will be sufficient to have a facility with maximum capacity level 4 in site 7 at a reduced cost (i.e., $75.49\% \times 100 < 80$). Applying the same procedure to the remaining instances in Table 2, all the 10 inferior LDR instances in Table 1 are correctly identified by Heuristic 1; in addition to the instance $(20, 10, 1, 5)$ where the LDR solution is in-parity with the scenario solution. Table 3 details the heuristic iterations including the uncertainty nominal value variation (Δ^t), time and total capacity surplus ($\hat{Q}$). For all instances, except two, the heuristic terminated after two iterations. For the remaining two, it required three iterations. The maximum allowed infeasible scenarios parameter ϵ is set to 0.1%.

Table 2: LDR facility utilization levels at 99% VaR cost under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert, \lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	Optimal facility location and capacities *(i;k)*	l_{ik} at 99% VaR cost
5,5,1,5	(1;5)	(89.2)
10,5,1,5	(4;5)	(78.5)
15,5,1,5	(11;5)	(89.5)
20,5,1,5	(15;1),(16;5)	(29.9,100)
5,10,1,5	(1;4),(4;5)	(72.2,100)
10,10,1,5	(6;1),(7;5),(10;5)	(83.2,75.5,92.5)
15,10,1,5	(9;5),(12;5),(15;1)	(78.2,100,58.7)
20,10,1,5	(16;5),(17;5)	(100,73.9)
5,15,1,5	(1;5),(2;5),(3;1),(4;5)	(99.6,100,82.4,57.0)
10,15,1,5	(1;5),(5;5),(7;5),(8;1)	(100,73.8,100,49.1)
15,15,1,5	(4;5),(6;5),(10;5)	(68.0,100,100)
20,15,1,5	(4;5),(5;5),(11;5)	(94.3,73.6,100)
5,20,1,5	(1;5),(2;5),(3;2),(4;5),(5;5)	(100,100,81.4,49.4,100)
10,20,1,5	(1;5),(4;5),(6;5),(7;2),(10;5)	(100,97.3,97.4,42.1,70.7)
15,20,1,5	(2;1),(4;5),(5;5),(11;5),(12;5)	(0,100,60.2,100,95.3)
20,20,1,5	(5;5),(7;5),(10;5),(12;5),(17;2)	(83.6,83.9,100,73,91.8)

Table 3: LDR robustness Tuning Heuristic 1 iterations under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Iteration 1			Iteration 2			Iteration 3		
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	$\Delta^t(\%)$	Time (sec)	$\hat{Q}$	$\Delta^t(\%)$	Time (sec)	$\hat{Q}$	$\Delta^t(\%)$	Time (sec)	$\hat{Q}$
10,10,1,5	20	330.2	400	17.6	335.4	0			
15,10,1,5	20	336.3	400	14.2	364.6	0			
20,10,1,5	20	338.2	400	12.5	358.3	0			
5,15,1,5	20	346.8	400	17.6	345.7	400	9.8	372.7	0
10,15,1,5	20	338.8	400	13.2	399.7	0			
15,15,1,5	20	346.9	400	12.0	395.9	0			
20,15,1,5	20	350.5	400	12.0	393.4	0			
5,20,1,5	20	334.2	800	11.1	361.3	0			
10,20,1,5	20	356.7	800	11.1	394.1	0			
15,20,1,5	20	370.1	800	13.4	365.1	0			
20,20,1,5	20	343.4	400	18.3	385.6	400	12.6	388.6	0

Table 4 summarizes the LDR solution with Tuning Heuristic 1 for the identified instances. The solution gap with the scenario-based solution is closed. Interestingly, for the $(20, 10, 1, 5)$ instance, the LDR-based solution is superior by 1.8%. This comes at the expense of 12 infeasible scenarios (i.e., $\hat{\epsilon} = 0.012\%$) which is considered to be negligible. The LDR total computational burden mainly comes from the heuristic step. For example, for the $(20, 20, 1, 5)$ instance, the fraction spent in the heuristic step is equal to $1105.9/1128.2 = 98\%$. The total time exceeds that of the scenario solution for small instances, but it still exhibits a clear advantage for large instances where $|\mathcal{I}| \geq 15$ and $|\mathcal{J}| \geq 15$.

Table 4: LDR with Tuning Heuristic 1 and scenario solution under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert$,	Simulated total cost			LDR Heuristic 1		Total time (sec)		$\hat{\epsilon}$ (%)	
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	LDR	Scen.	δ(%)	Iter.s	Time (sec)	LDR	Scen.	LDR	Scen.
10,10,1,5	10918.1	10918.1	0	2	676.8	677.3	313.3	0	0
15,10,1,5	11237.7	11237.7	0	2	685.0	685.9	497.9	0	0
20,10,1,5	11813.0	12023.9	-1.8	2	705.1	706.4	1373.2	0.01	0
5,15,1,5	16918.3	16918.3	0	3	1039.6	1040.4	173.5	0.05	0.05
10,15,1,5	17080.4	17080.4	0	2	684.9	686.2	243.0	0	0
15,15,1,5	18901.0	18901.0	0	2	705.9	707.3	1339.6	0	0
20,15,1,5	15278.4	15278.6	0	2	716.3	719.3	4869.8	0	0
5,20,1,5	23636.6	23636.6	0	2	700.7	701.6	86.0	0	0
10,20,1,5	26759.3	26759.3	0	2	724.1	725.7	1263.5	0	0
15,20,1,5	20267.7	20327.9	-0.3	2	750.0	763.6	10802.9	0	0
20,20,1,5	25900.7	25900.7	0	3	1105.9	1128.2	6593.4	0	0

The case for the Robustness Tuning Heuristic 2 is slightly different. It does not anticipate a priori an opportunity to reduce the LDR robustness. It aggressively attempts to do that for all instances. If an improved solution was not found or the maximum allowed infeasible simulated scenarios ϵ is exceeded, the heuristic terminates. We set $\epsilon = 1\%$, to explore solutions not found by Heuristic 1, and reduce the size of $\mathcal{S}'$, within the heuristic, to 10,000. The solution, total time and $\hat{\epsilon}$ comparison between LDR with Tuning Heuristic 2 and scenario methods are shown in Table 5. The heuristic not only was able to close the gap with the scenario-based cost for the inferior instances, but also provides insights of a better solution quality for five instances, (e.g. $(10, 5, 1, 5)$) at the expense of a predefined risk level (i.e., ϵ). It is worth noting that the total time has been significantly reduced due to the reduction of $\mathcal{S}'$ size.

Table 5: LDR with Tuning Heuristic 2 and scenario solution under uniform distributed customer demands.

$\|\mathcal{I}\|, \|\mathcal{J}\|,$	Simulated total cost			Total time (sec)		$\hat{\epsilon}$ (%)	
$\|\mathcal{P}\|, \|\mathcal{K}_i\|$	LDR	Scen.	δ(%)	LDR	Scen.	LDR	Scen.
5,5,1,5	5321.1	5321.1	0	52.2	22.1	0	0
10,5,1,5	4102.3	4321.8	-5.4	85.8	212.5	0.22	0
15,5,1,5	5138.2	5138.2	0	59.5	85.3	0	0
20,5,1,5	6856.1	6856.1	0	86.3	5196.0	0	0
5,10,1,5	8508.0	8756.2	-2.9	113.5	400.0	0.39	0
10,10,1,5	10918.1	10918.1	0	120.7	313.2	0	0
15,10,1,5	11237.7	11237.7	0	83.1	498.0	0	0
20,10,1,5	11815.3	12023.9	-1.8	77.9	1373.2	0	0
5,15,1,5	16918.3	16918.3	0	122.4	166.1	0.05	0.05
10,15,1,5	17080.4	17080.4	0	88.0	243.0	0	0
15,15,1,5	18901.1	18901.0	0	82.7	1339.6	0	0
20,15,1,5	15278.6	15278.6	0	85.2	4869.8	0	0
5,20,1,5	23636.6	23636.6	0	127.1	86.0	0	0
10,20,1,5	26759.3	26759.3	0	86.0	1263.5	0	0
15,20,1,5	20019.9	20327.9	-1.5	135.4	10802.9	0.27	0
20,20,1,5	25680.8	25900.7	-0.9	147.9	6593.4	0.45	0

Beta distributed customer demands

The second customer demand distribution investigated is a right-skewed beta distribution with $\alpha = 8$ and $\beta = 2$. We empirically show that the likelihood of overly-conservative LDR location decisions depends on the skewness of the distribution. The more it is skewed to the right (i.e., towards the upper bound of the demand range), the less the likelihood is. The results of the 16 instances are shown in Table 6 where the LDR solution is in-parity with the scenario solution in 11 instances in comparison to six instances under uniform distribution.

Table 6: LDR- and scenario-based results under beta distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$ $\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	Location cost			Simulated total cost			Time (sec)		$\hat{\epsilon}$ (%)	
	LDR	Scen.	δ(%)	LDR	Scen.	δ(%)	LDR	Scen.	LDR	Scen.
5,5,1,5	2184.6	2184.6	0	5697.4	5697.4	0	0.03	19.1	0	0
10,5,1,5	2028.6	2028.6	0	4597.9	4597.9	0	0.05	98.7	0	0
15,5,1,5	2150.9	2150.9	0	5496.9	5496.9	0	0.11	88.3	0	0
20,5,1,5	3492.0	3153.9	9.7	7309.3	7311.5	0	0.20	6456.9	0	0
5,10,1,5	3860.2	3860.2	0	9408.4	9408.4	0	0.08	331.3	0	0
10,10,1,5	5511.6	4383.0	20.5	12772.3	11706.8	8.3	0.25	98.0	0	0
15,10,1,5	5380.5	5380.5	0	13071.2	13071.2	0	0.55	10802.0	0	0
20,10,1,5	4200.4	4200.4	0	12988.3	12988.3	0	0.36	1265.8	0	0
5,15,1,5	8966.2	7286.3	18.7	19897.1	18323.1	7.9	0.30	76.3	0	0
10,15,1,5	8136.0	8136.0	0	19327.5	19327.5	0	0.39	3555.6	0	0
15,15,1,5	6452.5	6452.5	0	20666.3	20666.3	0	0.52	5158.6	0	0
20,15,1,5	6416.3	6416.3	0	16610.1	16610.1	0	0.97	10802.1	0	0
5,20,1,5	11893.4	11616.5	2.3	27204.4	26952	0.9	0.38	245.2	0	0
10,20,1,5	10459.9	10155.3	2.9	30358.5	30053.8	1.0	0.56	649.6	0	0
15,20,1,5	9667.3	8647.1	10.6	22977	21967.2	4.4	1.23	2086.8	0	0
20,20,1,5	10097.9	10012.8	0.8	29424.8	29463.5	-0.1	3.80	10802.9	0	0

Table 7 lists the facility utilization level at the 99% VaR simulated cost scneario for all instances. LDR Tuning Heuristic 1 identifies four out of five inferior instances in Table 6. The $(10, 10, 1, 5)$ instance is missed. Looking closely at its facility utilization levels, we observe that none of the facilities reached 100% utilization; the maximum is 91.7%. This is a side-effect of the simulator's effort to reduce the inferiority of the location decisions via the second-stage decisions (i.e., number of modules and flow amounts). It is worth noting that in all identified inferior instances for both distributions (16 out of 17), at least one of the facilities had full saturation at the 99% VaR cost scenario which is expected from an operational and practical point of view. The heuristic required two iterations for the identified instances as detailed in Table 8. The outcome is given in Table 9 where all LDR solution is in-parity with the scenario solution, despite the added computational cost for small instances where $|\mathcal{I}| \leq 10$. The size of $\mathcal{S}'$ within the heuristic is 100,000 and $\epsilon = 0.1\%$.

Table 7: LDR facility utilization levels at 99% VaR cost under beta distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert, \lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	Optimal facility location and capacities *(i;k)*	l_{ik} at 99% VaR cost
5,5,1,5	(1;5)	(93.1)
10,5,1,5	(4;5)	(81.6)
15,5,1;5	(11;5)	(93.1)
20,5,1,5	(15;1),(16;5)	(100,90.7)
5,10,1,5	(1;4),(4;5)	(84.4,100)
10,10,1,5	(6;1),(7;5),(10;5)	(87.5,86.7,91.7)
15,10,1,5	(9;5),(12;5),(15;1)	(90.1,100,58.7)
20,10,1,5	(16;5),(17;5)	(100,84.3)
5,15,1,5	(1;5),(2;5),(3;1),(4;5)	(100,100,86.5,75.3)
10,15,1,5	(1;5),(5;5),(7;5),(8;1)	(100,93.0,100,56.6)
15,15,1,5	(4;5),(6;5),(10;5)	(87.1,100,100)
20,15,1,5	(4;5),(5;5),(11;5)	(100,87.4,100)
5,20,1,5	(1;5),(2;5),(3;2),(4;5),(5;5)	(100,100,87.0,77.1,100)
10,20,1,5	(1;5),(4;5),(6;5),(7;2),(10;5)	(100,100,100,35.2,97.9)
15,20,1,5	(2;1),(4;5),(5;5),(11;5),(12;5)	(84.3,100,66.9,100,100)
20,20,1,5	(5;5),(7;5),(10;5),(12;5),(17;2)	(100,86.5,100,80.5,99.1)

Table 8: LDR Robustness Tuning Heuristic 1 iterations under beta distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Iteration 1			Iteration 2		
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	$\Delta^t(\%)$	Time (sec)	$\hat{Q}$	$\Delta^t(\%)$	Time (sec)	$\hat{Q}$
10,10,1,5	20	320.6	400	17.6	348.1	0
5,15,1,5	20	347.2	400	17.6	356.3	0
10,20,1,5	20	351.2	400	16.7	389.1	0
15,20,1,5	20	378.8	400	18.3	379.4	0

Table 9: LDR with Tuning Heuristic 1 and scenario solution under beta distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Simulated total cost			LDR Heuristic 1		Total time (sec)		$\hat{\epsilon}$ (%)	
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	LDR	Scen.	$\delta(\%)$	Iter.s	Time (sec)	LDR	Scen.	LDR	Scen.
10,10,1,5	11706.8	11706.8	0	2	641.2	641.6	98.0	0	0
5,15,1,5	18323.1	18323.1	0	2	694.4	694.9	76.3	0	0
10,20,1,5	30053.8	30053.8	0	2	702.5	703.6	649.6	0	0
15,20,1,5	21967.2	21967.2	0	2	757.5	760.9	2086.8	0	0

The results using the LDR Tuning Heuristic 2, $\hat{\epsilon} = 1\%$ and $|\mathcal{S}'| = 10{,}000$, are similar as shown in Table 10. In fact, the heuristic's aggressiveness is able to eliminate the LDR inferior quality in instance $(10, 10, 1, 5)$ which was not detected by Tuning Heuristic 1. The LDR risk levels for all instances, except one, is found to be zero. The aggressiveness of the heuristic may be increased to explore solutions with higher risk levels by reducing the VaR confidence value at which the facility utilization levels are computed (e.g., from 99% to 95%). This feature was not investigated in this work.

Table 10: LDR with Tuning Heuristic 2 and scenario solution under beta distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Simulated total cost			Total time (sec)		$\hat{\epsilon}$ (%)	
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	LDR	Scen.	$\delta(\%)$	LDR	Scen.	LDR	Scen.
5,5,1,5	5697.4	5697.4	0	57.1	19.1	0	0
10,5,1,5	4597.9	4597.9	0	61.6	98.7	0	0
15,5,1,5	5496.9	5496.9	0	60.1	88.3	0	0
20,5,1,5	7311.5	7311.5	0	119.4	6456.9	0	0
5,10,1,5	9408.4	9408.4	0	81.6	331.3	0	0
10,10,1,5	11706.8	11706.8	0	123.5	98.0	0	0
15,10,1,5	13071.2	13071.2	0	86.2	10802.0	0	0
20,10,1,5	12988.3	12988.3	0	83.8	1265.8	0	0
5,15,1,5	18323.1	18323.1	0	83.3	76.3	0	0
10,15,1,5	19327.5	19327.5	0	86.3	3555.6	0	0
15,15,1,5	20666.3	20666.3	0	83.0	5158.6	0	0
20,15,1,5	16610.1	16610.1	0	85.3	10802.1	0	0
5,20,1,5	26952.0	26952.0	0	86.8	245.2	0	0
10,20,1,5	30053.8	30053.8	0	86.7	649.6	0	0
15,20,1,5	21967.2	21967.2	0	83.3	2086.8	0	0
20,20,1,5	29463.5	29463.5	0	125.5	10802.9	0	0

Additional numerical instances are included in Appendix A. Out of the 64 instances, the LDR-based solution was found to be in-parity with the scenarios-based solution in 54 (i.e, 84.4%), superior in one (i.e., 1.6%), and inferior in nine instances (i.e., 14%). Out of the nine inferior instances, the LDR Heuristic 1 identified only two for further improvements. This is a pitfall for the heuristic's efficiency due to (i) the simulator's side-effect in reducing the sub-optimal location decisions via the adaptive number of modules and flow amounts decisions and (ii) the computational setting such as capacity levels. Nonetheless, the LDR Tuning Heuristic 2 is empirically shown to eliminate the LDR inferiority in all obtained instances and offers the opportunity to select an improved solution at an acceptable risk level for others. Yet, the aggressiveness of Heuristic 2 may have traversal effects as will be shown in the industrial instance later.

The best choice of the heuristic type depends on the instance where Heuristic 1 is preferred as a starting option. The decision maker may decide to use a hybrid strategy in tuning the LDR robustness where in each iteration Tuning Heuristic 1 is first implemented. If it does not identify any possible improvements (i.e., propose a new Δ^t), then Tuning Heuristic 2 is used. The hybrid strategy was implemented for all the numerical instances within this section and Appendix Appendix A. The final solution is the same as that of Tuning Heuristic 2, but the Pareto front (solution quality vs $\hat{\epsilon}$) obtained along the search path to the final solution is different.

Overall, the numerical experiments shows that a solution which is in-par with the scenario-based approach can be obtained using LDR with robustness tuning heuristics at a lower computational cost. They also demonstrate that for some cases, an attractive trade-off between solution quality and risk is achieved.

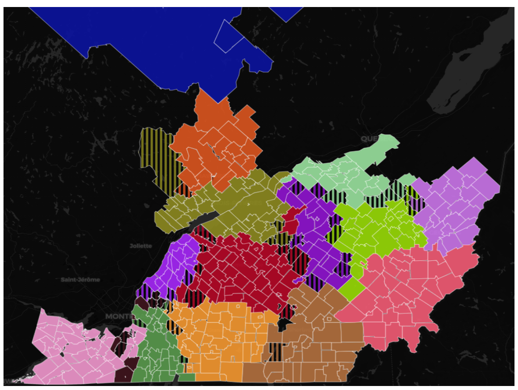
5.2. Industrial case study

The industrial instance provided by Hydro-Québec includes 17 sites, 474 customers, a single service product and four maximum capacity levels at 5, 10, 15 and 20 service teams or modules (i.e., $|\mathcal{I}|, |\mathcal{J}|, |\mathcal{P}|, |\mathcal{K}_i| = 17, 472, 1, 4$). We assume that the initial number of service teams in all sites i is zero (i.e., no existing facilities). The customers' uncertain demands vary within 20% of the given nominal values. Two demand distributions are explored independently: a uniform distribution and a beta ($\alpha = 8$, $\beta = 2$) distribution. All customers' uncertain demands are assumed to be independent and follow the same distribution. As per company policy, Hydro-Québec requires that the transportation costs for customers that are outside a given radius from a service center (i.e., a site) are heavily penalized to reduce such occurrences to the minimum. Each customer demand is ideally modelled as a distinct uncertain parameter and the dimension of the uncertainty space is equal to the number of customers. However, the 474 uncertain parameters, having the same uncertainty distribution, may be aggregated into a single parameter in the look-ahead model to compute the location decisions. Hence, we emphasize that the uncertain parameters are aggregated in the model, yet they are distinct and *not* aggregated in the simulator.

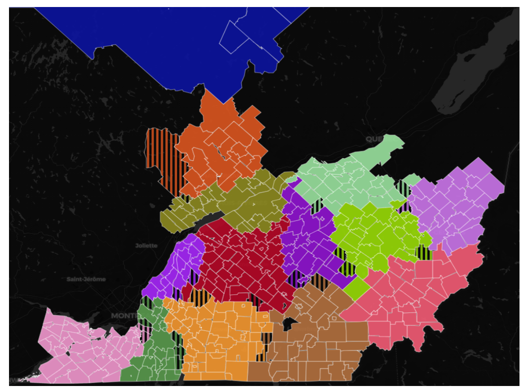
Table 11 shows the LDR results breakdown with and without tuning heuristics for beta and uniform distributions. For both distributions, the LDR location decisions without tuning heuristic suggests opening service centers in 15 out of the 17 sites with a location cost of 267.59 million dollars and a total of 135 service teams in the network. Applying Tuning Heuristics 1 and 2 under beta distribution, the location costs decrease by 2.50% and 4.93%, respectively. However, the expected total cost reduction is equal to 1.89% and 1.37%, respectively. For Tuning Heuristic 2, the optimal number of service centers reduces to 14. The aggressiveness of Tuning Heuristic 2 in reducing the inferiority of the location decisions comes at the expense of a non-anticipated higher mean transportation costs. On the other hand, the LDR Heuristic 1 outcome's decision to pay more now (i.e., location costs) created the opportunity to save more later (i.e., mean transportation costs) and performs better overall. Under the uniform distribution, LDR Heuristics 1 and 2 solution improved the mean total cost by 3.93% and 5.20%, respectively. For both tuning heuristics, the number of optimal service centers reduces to 14. Though both LDR heuristic solutions have the same total number of service teams, the capacity distribution among different sites differ with LDR Heuristic 2 location costs at a 2% advantage. Yet, the LDR Heuristic 2 outcome led to a less "regular" service center-to-customer allocation as shown in Figure 2. The optimal geographical map, based on mean values, illustrates the number opened service centers and the customers served by each, in a distinct color, for both distributions and robustness tuning heuristics. The shaded region indicates a customer served by more than one service center.

Table 11: LDR solution with and without robustness tuning heuristics for the industrial instance.

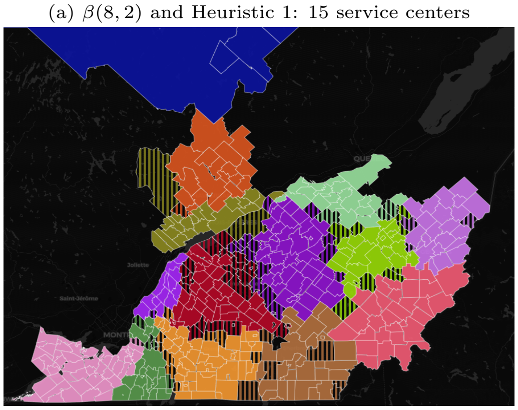
	$\beta(8,2)$ distribution			$\mathcal{U}$ distribution		
Tuning heuristic $\rightarrow$	None	1	2	None	1	2
Location cost (Millions)	267.59	260.90	254.40	267.59	247.76	242.27
Total number of service teams	135	130	125	135	120	120
Mean transportation cost (Millions)	100.02	99.77	108.18	85.86	91.79	92.81
Mean total cost (Millions)	367.61	360.67	362.58	353.45	339.55	335.08
Location cost change (%)	—	-2.50	-4.93	—	-7.41	-9.46
Mean total cost change (%)	—	-1.89	-1.37	—	-3.93	-5.20



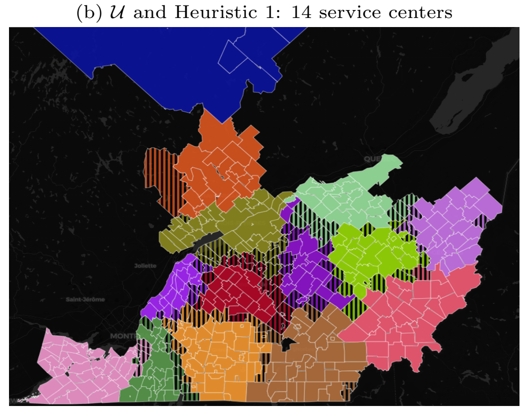
(a) $\beta(8,2)$ and Heuristic 1: 15 service centers



(b) $\mathcal{U}$ and Heuristic 1: 14 service centers



(c) $\beta(8,2)$ and Heuristic 2: 14 service centers



(d) $\mathcal{U}$ and Heuristic 2: 14 service centers

Figure 2: Located service centers and customers based on mean values for both distributions and tuning heuristics.

A criterion sought after by the company which is not explicitly defined in the model is the site-to-customer allocation. Ideally, a logistics network where each customer is served by a site or a single service center is preferred. In practice, the more customers are served by only one service center in all scenarios, the better. The distribution of customers served by only one, up to two or up to three service centers using both heuristics under both distributions are shown in Table 12. In particular, under the uniform distribution, we observe that 94% of customers are always served by one service center using Tuning Heuristic 1 while the measure is only 76.2% using Tuning Heuristic 2. Figure 3 visualizes, for three illustrative customers served by one or more service centers, the number of service centers under beta distribution and using Robustness Tuning Heuristic 1. In the cases where a customer may require up to three service centers to fulfill its uncertain demand, the percentage where all three service centers are needed is found to be negligible as in Figure 3c. We developed a similar utility to visualize the number of customers distribution served by a given service center. Figure 4 illustrates the examples of illustrative service centers using Robustness Tuning Heuristic 1 under a beta and uniform distributions, respectively.

Table 12: Distribution of customers served by a given number of service centers.

	$\beta(8,2)$ distribution		$\mathcal{U}$ distribution	
Tuning heuristic $\rightarrow$	1	2	1	2
Only one service center	412	393	447	361
Up to two service centers	58	73	27	96
Up to three service centers	4	8	0	17

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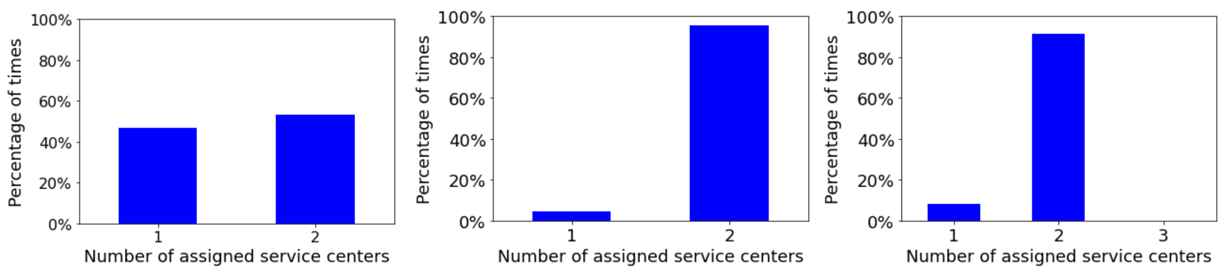
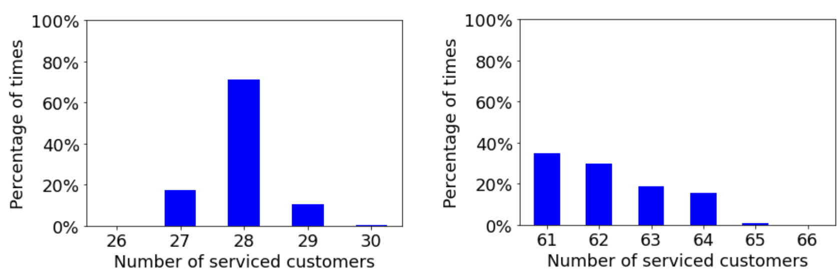


(a) 1 and 2 service centers

(b) 1 and 2 service centers

(c) 1, 2 and 3 service centers

Figure 3: Service centers distribution for three illustrative customers under a beta distributed demands and using Robustness Tuning Heuristic 1.



(a) Under $\beta(8,2)$

(b) Under $\mathcal{U}$

Figure 4: Customers number distribution for two illustrative service centers using Robustness Tuning Heuristic 1.

Lastly, the current and optimal service centers' capacities under $\beta(8,2)$ and $\mathcal{U}$ distributions using LDR Tuning Heuristic 1 are shown in Table 13. Assuming the beta distributed customer demands, the solution suggests closing two service centers, operating five of the remaining 15 service centers at a 25% overcapacity.

As for the uniform distributed customer demands assumption, the solution proposes closing three service centers, operating four of the remaining 14 service centers at a 25% overcapacity.

Table 13: Current and suggested service centers' capacities under $\beta(8, 2)$ and $\mathcal{U}$ distributions using LDR tuning Heuristic 1.

Site index	Current	Optimal	
		$\beta(8, 2)$	$\mathcal{U}$
1	12	0	0
2	5	5	5
3	10	10	10
4	17	0	0
5	14	5	5
6	17	20	20
7	12	15	15
8	15	15	15
9	11	5	0
10	8	10	10
11	8	5	5
12	2	5	5
13	14	15	10
14	9	5	5
15	4	5	5
16	8	5	5
17	10	5	5

6. Conclusions

In this paper, we have considered a facility location problem with modular capacity under demand uncertainty faced by Hydro-Québec, an industrial collaborator from the public service sector. We have developed a mathematical model and have presented a two-stage stochastic framework to address the uncertainty. We have investigated the stochastic scenario-based and linear decision rule-based methods where the competitive advantage of the latter in terms of computational cost has been emphasized through a set of numerical computational experiments.

Due to the problem's nature, cost minimization with a strict constraint to satisfy demand, linear decision rule location decisions may be unnecessarily conservative. We have integrated a feedback mechanism into a linear decision solution method to tune the LDR robustness at the expense of manageable risk. The mechanism implicitly incorporates information about the demand distribution into the model via the value at risk measure and facilities utilization levels via a truncated uncertainty set. Two heuristic algorithms have been proposed. The benefits have been empirically shown through a set of numerical experiments in addition to the industrial instance.

This work paves the way for future research directions. Linear decision rules stand as an attractive instrument to solve multistage facility location problem under uncertainty due to their competitive algorithmic features. In contrast to scenario-based solution method, it does not suffer the curse of dimensionality. Further, the approximation of the adaptive/recourse binary decisions as linear decision rules in a facility location context is a direction yet to be explored. Lastly, coupling LDR method with decomposition techniques or valid inequalities enforces the method's attractiveness in a multistage context.

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Appendix A. Additional computational numerical instances

To further explore the LDR robustness tuning heuristics' efficiency, we extend the cases of the available maximum capacities $|\mathcal{K}_i|$ for all sites to 3 and 5. Tables A.14 and A.16 show the results for the uniform and beta ($\alpha = 8$, $\beta = 2$) distributions using Robustness Tuning Heuristic 1, respectively.

Under uniformly distributed demands, the LDR-based solution is in-parity with the scenarios-based solution in 27 out of 32 instances. Two out of the five LDR inferior instances are detected by the Tuning Heuristic 1. The final output after one iteration is shown in Table A.15. Under the beta distributed demands, three out of 32 instances are found inferior. Nonetheless, none of the cases are detected by the heuristic. The outcome emphasizes a limitation for the Tuning Heuristic 1 resulting from the simulator's side-effect of reducing the location decisions inferiority. Nonetheless, Robustness Tuning Heuristic 2 does not suffer from this limitation. It is able to eliminate the inferiority in all found instances and it provides a possible attractive trade-off between cost and risk for others. The LDR with Robustness Tuning Heuristic 2 results for both distributions are shown in Tables A.17 and A.18.

Table A.14: Additional LDR- and scenario-based results under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$ $\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	Location cost			Simulated total cost			Time (sec)		$\hat{\epsilon}$ (%)	
	LDR	Scen.	δ(%)	LDR	Scen.	δ(%)	LDR	Scen.	LDR	Scen.
5,5,1,1	2060.3	2060.3	0	5673.1	5673.1	0	0.05	27.5	0	0
10,5,1,1	2001.6	2001.6	0	5819.0	5819.0	0	0.06	40.0	0	0
15,5,1,1	960.0	960.0	0	2950.5	2950.5	0	0.09	6.4	0	0
20,5,1,1	1093.5	1093.5	0	4306.1	4306.1	0	0.11	13.4	0	0
5,10,1,1	3376.4	3376.4	0	10515.2	10515.2	0	0.06	36.3	0	0
10,10,1,1	2355.0	2355.0	0	7459.3	7459.3	0	0.22	663.8	0	0
15,10,1,1	2481.9	2481.9	0	8414.2	8414.2	0	0.36	957.5	0	0
20,10,1,1	3347.5	3347.5	0	12709.8	12709.8	0	0.28	2766.7	0	0
5,15,1,1	3668.2	3668.2	0	12112.3	12112.3	0	0.16	51.2	0	0
10,15,1,1	3817.9	3817.9	0	13907.4	13907.4	0	0.16	843.6	0	0
15,15,1,1	4756.9	4756.9	0	19101.6	19101.6	0	0.42	1538.5	0	0
20,15,1,1	4790.2	4790.2	0	19811.1	19811.1	0	1.39	2034.9	0	0
5,20,1,1	6041.1	6041.1	0	19431.5	19431.5	0	0.09	56.9	0	0
10,20,1,1	7069.6	7069.6	0	25753.1	25753.1	0	0.44	395.7	0	0
15,20,1,1	6161.5	6161.5	0	25770.0	25770.0	0	0.53	830.5	0	0
20,20,1,1	5580.0	5580.0	0	20453.4	20453.4	0	1.77	3650.5	0	0
5,5,1,3	2941.6	2941.6	0	6760.0	6760.0	0	0.05	23.5	0	0
10,5,1,3	1661.7	1661.7	0	4302.3	4302.3	0	0.06	24.6	0	0
15,5,1,3	2819.4	2819.4	0	7866.6	7866.6	0	0.25	1074.9	0	0
20,5,1,3	2612.9	1593.3	39.0	6979.7	5967.8	14.5	0.25	132.3	0	0.01
5,10,1,3	4473.2	4473.2	0	10321.6	10321.6	0	0.09	175.5	0	0
10,10,1,3	4994.9	4691.9	6.0	14883.0	14579.9	2.0	0.27	623.7	0	0
15,10,1,3	3199.4	3199.4	0	10872.2	10872.2	0	0.19	6622.6	0	0
20,10,1,3	4096.6	4096.6	0	9830.7	9830.7	0	1.28	7200.4	0	0
5,15,1,3	8081.9	8081.9	0	23834.4	23834.4	0	0.19	232.5	0	0
10,15,1,3	5325.1	5325.1	0	17626.6	17626.6	0	0.45	2961.8	0	0
15,15,1,3	6167.4	4891.4	20.7	14361.5	13262.6	7.7	1.70	2223.2	0	0.07
20,15,1,3	6745.7	5355.0	20.6	18331.7	17069.2	6.9	4.11	5822.0	0	0
5,20,1,3	7522.7	7522.7	0	24246.8	24246.8	0	0.27	188.1	0	0
10,20,1,3	7949.8	6597.9	17.0	18278.3	17252.3	5.6	1.77	4067.3	0	0
15,20,1,3	7843.6	7843.6	0	21937.5	21937.5	0	3.11	7200.7	0	0
20,20,1,3	9268.9	8612.7	7.1	29054.3	28889.0	0.6	4.45	7200.7	0	0

Table A.15: Additional LDR with Robustness Tuning Heuristic 1 iterations under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$ $\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	Simulated total cost			LDR Heuristic 1		Total time (sec)		$\hat{\epsilon}$ (%)	
	LDR	Scen.	δ(%)	Iters.	Time (sec)	LDR	Scen.	LDR	Scen.
10,10,1,3	4691.9	4691.9	0	1	734.0	734.5	623.7	0	0
15,15,1,3	14085.3	13262.6	5.8	1	779.2	783.0	2223.2	0	0

Table A.16: Additional LDR- and scenario-based results under beta distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Location cost			Simulated total cost			Time (sec)		$\hat{\epsilon}$ (%)	
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	LDR	Scen.	δ(%)	LDR	Scen.	δ(%)	LDR	Scen.	LDR	Scen.
5,5,1,1	2060.3	2060.3	0	6105.9	6105.9	0	0.05	30.8	0	0
10,5,1,1	2001.6	2001.6	0	6277.3	6277.3	0	0.06	27.7	0	0
15,5,1,1	960.0	960.0	0	3190.0	3190.0	0	0.06	3.8	0	0
20,5,1,1	1093.5	1093.5	0	4692.8	4692.8	0	0.08	18.3	0	0
5,10,1,1	3376.4	3376.4	0	11372.3	11372.3	0	0.06	51.6	0	0
10,10,1,1	2355.0	2355.0	0	8081.0	8081.0	0	0.17	937.6	0	0
15,10,1,1	2481.9	2481.9	0	9135.0	9135.0	0	0.22	582.5	0	0
20,10,1,1	3347.5	3347.5	0	13833.8	13833.8	0	0.30	1533.8	0	0
5,15,1,1	3668.2	3668.2	0	13151.2	13151.2	0	0.14	25.7	0	0
10,15,1,1	3817.9	3817.9	0	15157.8	15157.8	0	0.17	143.5	0	0
15,15,1,1	4756.9	4756.9	0	20844.3	20844.3	0	0.48	1068.5	0	0
20,15,1,1	4790.2	4790.2	0	21611.7	21611.7	0	0.89	2421.6	0	0
5,20,1,1	6041.1	6041.1	0	21071.1	21071.1	0	0.13	57.9	0	0
10,20,1,1	7069.6	7069.6	0	27997.7	27997.7	0	0.38	528.4	0	0
15,20,1,1	6161.5	6161.5	0	28142.9	28142.9	0	0.48	809.4	0	0
20,20,1,1	5580.0	5580.0	0	22330.0	22330.0	0	1.75	4531.7	0	0
5,5,1,3	2941.6	2941.6	0	7224.8	7224.8	0	0.05	12.7	0	0
10,5,1,3	1661.7	1661.7	0	4619.3	4619.3	0	0.08	24.9	0	0
15,5,1,3	2819.4	2819.4	0	8471.5	8471.5	0	0.55	1021.3	0	0
20,5,1,3	2612.9	2612.9	0	7503.2	7503.2	0	0.19	1459.9	0	0
5,10,1,3	4473.2	4473.2	0	11094.6	11094.6	0	0.09	256.4	0	0
10,10,1,3	4994.9	4691.9	6.1	16074.9	15771.8	1.9	0.53	417.4	0	0
15,10,1,3	3199.4	3199.4	0	11794.1	11794.1	0	0.45	7201.2	0	0
20,10,1,3	4129.1	4129.1	0	10544.2	10544.2	0	1.03	7200.3	0	0
5,15,1,3	8081.9	8081.9	0	25789.2	25789.2	0	0.20	95.8	0	0
10,15,1,3	5325.1	5325.1	0	19119.2	19119.2	0	0.33	789.6	0	0
15,15,1,3	6167.4	5889.4	4.5	15347.3	15069.3	1.8	1.38	7201.0	0	0
20,15,1,3	6926.8	6792.0	1.9	19740.8	19798.5	-0.3	2.83	7200.6	0	0
5,20,1,3	7522.7	7522.7	0	26260.9	26260.9	0	0.20	82.5	0	0
10,20,1,3	7949.8	6597.9	17.0	19520.6	18632.8	4.5	1.38	372.5	0	0.01
15,20,1,3	7843.6	7843.6	0	23640.1	23640.1	0	1.41	4983.0	0	0
20,20,1,3	9268.9	9268.9	0	31445.7	31445.7	0	3.14	7200.4	0	0

Table A.17: Additional LDR with Robustness Tuning Heuristic 2 and scenario solution under uniform distributed customer demands.

$\lvert\mathcal{I}\rvert, \lvert\mathcal{J}\rvert,$	Simulated total cost			Total time (sec)		$\hat{\epsilon}$ (%)	
$\lvert\mathcal{P}\rvert, \lvert\mathcal{K}_i\rvert$	LDR	Scen.	$\delta(\%)$	LDR	Scen.	LDR	Scen.
5,5,1,1	5673.1	5673.1	0	58.2	27.5	0	0
10,5,1,1	5819.0	5819.0	0	59.3	40.0	0	0
15,5,1,1	2950.5	2950.5	0	48.5	6.4	0	0
20,5,1,1	4306.1	4306.1	0	54.6	13.4	0	0
5,10,1,1	10515.2	10515.2	0	69.2	36.3	0	0
10,10,1,1	7459.3	7459.3	0	70.2	663.8	0	0
15,10,1,1	8414.2	8414.2	0	70.9	957.5	0	0
20,10,1,1	12709.8	12709.8	0	70.2	2766.7	0	0
5,15,1,1	12112.3	12112.3	0	74.1	51.2	0	0
10,15,1,1	13907.4	13907.4	0	76.8	843.6	0	0
15,15,1,1	18052.0	19101.6	-5.8	133.3	1538.5	0.96	0
20,15,1,1	19811.1	19811.1	0	94.3	2034.9	0	0
5,20,1,1	19431.5	19431.5	0	80.0	56.9	0	0
10,20,1,1	24412.7	25753.1	-5.5	124.8	395.7	0.12	0
15,20,1,1	24522.7	25770.0	-5.1	130.7	830.5	0.64	0
20,20,1,1	20453.4	20453.4	0	91.4	3650.5	0	0
5,5,1,3	6760.0	6760.0	0	68.6	23.5	0	0
10,5,1,3	4302.3	4302.3	0	55.7	24.6	0	0
15,5,1,3	7866.6	7866.6	0	77.5	1074.9	0	0
20,5,1,3	5967.8	5967.8	0	69.8	132.3	0	0
5,10,1,3	9294.1	10321.6	-11.1	74.5	175.5	0.14	0
10,10,1,3	14579.9	14579.9	0	76.5	623.7	0	0
15,10,1,3	10584.6	10872.2	-2.7	119.5	6622.6	0.24	0
20,10,1,3	8974.0	9830.7	-9.5	118.1	7200.4	0.09	0
5,15,1,3	23535.1	23834.4	-1.3	114.3	232.5	0.03	0
10,15,1,3	17348.3	17626.6	-1.6	122.6	2961.8	0.37	0
15,15,1,3	13262.6	13262.6	0	79.4	2223.2	0	0
20,15,1,3	17069.2	17069.2	0	85.2	5822.0	0	0
5,20,1,3	24007.3	24246.8	-1.0	75.9	188.1	0.01	0
10,20,1,3	17252.3	17252.3	0	128.8	4067.3	0	0
15,20,1,3	21705.3	21937.5	-1.1	138.3	7200.7	0.02	0
20,20,1,3	28789.3	28889.0	-0.3	106.9	7200.7	0	0

Table A.18: Additional LDR with Robustness Tuning Heuristic 2 and scenario solution under beta distributed customer demands.

$\|\mathcal{I}\|, \|\mathcal{J}\|,$	Simulated total cost			Total time (sec)		$\hat{\epsilon}$ (%)	
$\|\mathcal{P}\|, \|\mathcal{K}_i\|$	LDR	Scen.	δ(%)	LDR	Scen.	LDR	Scen.
5,5,1,1	6105.9	6105.9	0	61.2	30.8	0	0
10,5,1,1	6277.3	6277.3	0	65.2	27.7	0	0
15,5,1,1	3190.0	3190.0	0	58.8	3.8	0	0
20,5,1,1	4692.8	4692.8	0	61.8	18.3	0	0
5,10,1,1	11372.3	11372.3	0	66.7	51.6	0	0
10,10,1,1	8081.0	8081.0	0	72.2	937.6	0	0
15,10,1,1	9135.0	9135.0	0	74.8	582.5	0	0
20,10,1,1	13833.8	13833.8	0	79.3	1533.8	0	0
5,15,1,1	13151.2	13151.2	0	66.9	25.7	0	0
10,15,1,1	15157.8	15157.8	0	71.0	143.5	0	0
15,15,1,1	20844.3	20844.3	0	78.7	1068.5	0	0
20,15,1,1	21611.7	21611.7	0	87.0	2421.6	0	0
5,20,1,1	21071.1	21071.1	0	74.1	57.9	0	0
10,20,1,1	27997.7	27997.7	0	82.4	528.4	0	0
15,20,1,1	28142.9	28142.9	0	86.4	809.4	0	0
20,20,1,1	22330.0	22330.0	0	81.1	4531.7	0	0
5,5,1,3	7224.8	7224.8	0	59.0	12.7	0	0
10,5,1,3	4619.3	4619.3	0	50.3	24.9	0	0
15,5,1,3	8471.5	8471.5	0	72.2	1021.3	0	0
20,5,1,3	6490.6	7503.2	-15.6	68.2	1459.9	0.07	0
5,10,1,3	11094.6	11094.6	0	66.6	256.4	0	0
10,10,1,3	15771.8	15771.8	0	73.5	417.4	0	0
15,10,1,3	11794.1	11794.1	0	76.8	7201.2	0	0
20,10,1,3	10544.2	10544.2	0	82.7	7200.3	0	0
5,15,1,3	25789.2	25789.2	0	72.7	95.8	0	0
10,15,1,3	19119.2	19119.2	0	74.7	789.6	0	0
15,15,1,3	15069.3	15069.3	0	84.6	7201.0	0	0
20,15,1,3	19732.0	19798.5	-0.3	90.1	7200.6	0	0
5,20,1,3	26260.9	26260.9	0	74.6	82.5	0	0
10,20,1,3	18632.8	18632.8	0	81.1	372.5	0	0
15,20,1,3	23640.1	23640.1	0	80.7	4983.0	0	0
20,20,1,3	31209.2	31445.7	-0.8	98.0	7200.4	0.15	0

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