Multi-objective Maritime Vessel Routing with Safety Considerations

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Abstract

The routing of maritime vessels is a challenging optimization problem that often involves finding an adequate balance between multiple objectives. This paper proposes a methodology based on inverse optimization to find appropriate objective weights that account for conflicting objectives, multiple safety considerations, route time, and fuel consumption. The motivation behind our choice of approach lies in the complexity of determining objective weights in multi-objective problems and the need for incorporating the preferences of multiple stakeholders. To achieve this, our methodology relies on first generating a set of "best practice routes". These routes can be constructed based on expert knowledge, real-world weather information, and analysis of criteria as specified by domain experts. Within the scope of this study, these routes are generated using our optimization model, with predefined objective weights applied to evaluate the efficacy of the approach. To formulate the inverse optimization problem, we use dual programming and the Karush-Kuhn-Tucker (KKT) optimality conditions. Our approach incorporates safety aspects into the decision-making process encompassing dynamic stability, the probability of bow

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slamming and green water occurrences. The results demonstrate that the proposed inverse optimization model identifies the weights associated with the best practice routes. Additionally, the methodology yields routing decisions more aligned with these routes than the Analytic Hierarchy Process (AHP) method.

Keywords: Multi-objective routing problem, inverse optimization, weight estimation methods, vessel routing, maritime safety.

1 Introduction

A large part of international trade is made possible by maritime transport (Yan et al., 2021b). This mode of transportation covers all oceans and is used in all weather conditions ranging from calm water to violent hurricanes when vessel safety is threatened. Vessel weather route planning is complex, having to balance multiple objectives such as route time, fuel consumption and safety. The monetary cost of a route can be broken down into two primary components: route time and fuel consumption. In cargo shipping, route time costs are primarily assessed based on vessel utilization and operational expenses, including time charter rates and crew wages. For instance, when a vessel is chartered, it typically incurs a time-based charter rate, representing the cost of using the vessel for a specific period. As the journey duration increases, so does the time charter cost. Fuel cost constitutes an important portion of cargo vessel operating expenses, influenced by various factors, including fuel consumption, speed setting, weather condition, fuel prices, and compliance with environmental regulations controlling fuel type.

However, measuring safety requires a different approach than using only monetary values. Safety at sea is a primary concern for captains both before starting and during navigation. The weather information itself has limitations. Typically, weather forecasts are considered reliable for up to seven days, with a higher degree of uncertainty towards the end of this period. Additionally, some weather forecasts can be highly dynamic and uncertain. A notable example of this includes predictions regarding potential hurricanes or typhoons. Hence, there is a need to balance monetary cost with safety concerns. It encompasses protection from the risk of injury, navigational hazards, environmental and sea conditions. The measure of effectiveness in maritime navigation safety is determined by the vessel's characteristics, the weather conditions, and the skillset of the navigators collectively prevent accidents and unforeseen events on the water (Formela, 2019). In vessel route optimization problems, ensuring safety involves a combination of hard and soft constraints, each serving specific purposes. Strict adherence to hard constraints is essential to safeguard the vessel from potential hazards. These constraints act as preventive measures and prohibit navigation in areas with significant risks, such as capsizing, grounding, falling to piracy, entering war zones, or colliding with oil platforms. Soft constraints, on the other hand, consider both risk and comfort levels in routing problems. They are more flexible than hard constraints and may involve a penalty-based approach for non-compliance. Hard and soft constraints are dependent on the timely availability of quality information, including weather forecasts and vessel operations under different sea conditions. For instance, when dealing with hurricanes, a hard constraint states that the vessel must maintain a minimum distance defined by the hurricane's frontier in order to avoid capsizing, ensuring the crew's safety. Simultaneously, the soft constraint encourages increasing the distance from hurricanes, providing an additional layer of safety and comfort especially given the varying degrees of uncertainty in forecasts. In route optimization problems, hard constraints play a crucial role by restricting the solution space. Ideally, soft constraints should be met, but if violated, there is a cost to maintain feasibility. This can be accomplished by penalizing its violations in the objective function. Nevertheless, integrating hard safety constraints with monetary objectives adds another layer of complexity due to the distinct nature of value scales involved. Monetary values are often substantial, whereas safety is usually quantified through a risk function. This discrepancy makes it challenging to properly balance these objectives, necessitating advanced optimization techniques to ensure that neither cost efficiency nor safety is compromised. Furthermore, quantifying a variety of safety concerns adds to the complexity.

The International Maritime Organization (IMO) regulations play a vital role in reducing the environmental impact of maritime transportation, including mandatory restrictions on sulphur emissions (Joung et al., 2020). Compliance with these regulations is critical for both environmental sustainability and meeting global standards for maritime pollution control. IMO changed the rules in January 2020 for maritime transportation to reduce the sulphur content from a maximum of 3.5% to 0.5% (Joung et al., 2020). This enactment imposes higher costs related to fuel consumption. Moreover, IMO has developed and adopted international collision regulations and global standards for seafarers to ensure the safety of the voyages crossing the oceans (IMO, 1972). Hence, vessel route planning must adhere to the safety guidelines specified by IMO. These essential rules include various aspects to ensure the safe navigation of ships at sea that are explained in more detail in Section 2.2.

Considering the complex nature of the regulations and the associated increased compliance costs, it is vital for shipping companies to optimize their routes. This involves balancing time, fuel, and safety considerations. Finding the proper balance between these objectives is challenging.

When it comes to the priorities of vessel stakeholders, the balance between these objectives can vary. Increasing safety might lead to longer voyage route times and increased fuel consumption. Some stakeholders prioritize quick routes to accommodate priority loads, while others, like captains, might choose slower speeds to enhance safety and avoid hazards. Charterers typically aim for a compromise between cost-efficiency and the timely delivery of cargo. On the other hand, vessel owners often seek a balance that optimizes both ship utilization and fuel consumption while adhering to safety standards.

Multi-objective optimization is used in many practical applications. In practice, optimization problems can have two or more objectives worth being considered in the objective function. These objectives are usually in conflict as the improvement of one objective can deteriorate the others. A fundamental concept in such problems is the Pareto frontier of efficient solutions. It defines a specific subset of solutions, where it is impossible to improve one objective without negatively affecting at least one other objective (Coello, 2007). Cohon (1983) developed the concept of Pareto optimality, which is a significant contribution to the field of multi-objective optimization. The generation of the Pareto frontier follows a systematic procedure. For our specific problem, it goes as follows: First, objectives are identified. Then, an objective function is formulated by scalarizing the objectives with weights, reflecting their relative importance. Subsequently, all possible weight settings are established, and the optimization problem is solved to identify optimal solutions for each specific weight configuration. Non-dominated solutions, which cannot be improved upon across all objectives at once, are gathered. These solutions ultimately define the Pareto frontier, illustrating the trade-offs between objectives. Decision makers (DMs) can then make informed choices from the frontier based on their preferences.

A classical method to deal with multi-objective optimization problems is the weighted sum method that transforms multiple objective problems into a single one by scalarizing the objectives with weights reflecting the judgment of the DMs. This method was first introduced by Zadeh (1963). Since then, the literature on the applications of the method has been growing steadily (see e.g., the reviews by Cohon, 1983, and by Odu and Charles-Owaba, 2013). The weighted method enables DMs to customize the prioritization of diverse objectives according to their individual preferences. It offers a combination of flexibility and simplicity in the optimization processes. Nevertheless, this approach brings about certain challenges, including subjectivity and the possibility of bias. It not only requires DMs to carefully analyze and balance trade-offs among various objectives, but it also must be applied within specific limitations or boundaries. These limitations imply that DMs are frequently operating under a set of boundaries, such as limited resources, time constraints, or other crucial factors, necessitating a strategic approach in navigating these limitations while assigning weights to different objectives. Although a weighted sum method can offer a systematic approach to multi-objective optimization, it is sensitive to the chosen weights. This implies that even small changes in the weights assigned to different objectives can result in significantly different outcomes, making it challenging to identify the precise set of weights that accurately represent the DM's preferences (Mavrotas, 2009). This sensitivity introduces decision-making challenges, such as inconsistent outcomes, inaccurate reflection of priorities, and risks of bias. Therefore, conducting a comprehensive evaluation becomes essential in order to thoroughly evaluate and ascertain the suitability and reliability of the weighted method within a specified context. Of the various techniques available, sensitivity analysis is the most common and crucial one. This process involves altering the weights for different objectives and observing the resulting changes in outcomes. Through this method, DMs can identify which objectives most significantly influence their decisions and assess the stability and reliability of these decisions under different weight scenarios.

Odu and Charles-Owaba (2013) present an overview of the principles and techniques of multi-criteria optimization, including both deterministic and stochastic approaches. By providing a detailed evaluation and comparison of various multi-criteria optimization methods, the article explores their relative advantages and disadvantages. Specifically, it discusses several methods in detail, including the weighted sum method, goal programming, and the Analytic Hierarchy Process (AHP). Goal programming (Charnes et al., 1955) is a method to solve multi-objective optimization problems. This approach aims to optimally fulfill the goals, emphasizing more critical ones and allowing for some trade-offs among less important objectives. The process involves setting target values for each goal and minimizing deviations from these targets. AHP (Saaty, 1980) is a powerful decision-making method developed by Thomas L. Saaty in the

1970s that has found widespread application in various fields. It offers a structured approach to address challenges like assigning weights in the weighted method. By breaking decisions down into a hierarchical structure, AHP systematically determines the relative importance of multiple criteria and alternatives through a multi-level framework. Inverse optimization, introduced by Burton and Toint (1992), is another decision-making approach that infers the underlying objectives or preferences of a DM by analyzing their observed behavior or the outcome of their behavior. The applications of this method in determining objective weights are vast, and we exemplify this through a description of several studies in the field. Ajayi et al. (2022) used inverse optimization to develop a method for selecting optimal clinical objectives for radiation therapy treatment planning of prostate cancer. This strategy aims to infer a set of objectives from historical treatment data by measuring and minimizing the deviation between observed treatments and potential optimal scenarios, thereby identifying objectives that accurately represent successful historical treatments. Terekhov et al. (2010) presents an inverse optimization approach with a focus on human prehension tasks. The main challenge addressed is determining the unknown objective function being optimized in human actions, particularly in the context of various motor tasks and movement aspects. Moreover, Togo et al. (2022) applied an inverse optimization approach methodology for estimating the weighting factors of the objective function in scheduling problems using historical data that includes operation time and setup costs. Gebken and Peitz (2021) detailing a method for constructing objective function vectors in multi-objective optimization problems based on given data points. The proposed approach addresses the inverse problem of multi-objective optimization by finding objective function vectors that make a given Pareto set or data points Pareto critical under certain conditions.

In this paper, we make three contributions to the field of vessel route planning. First, we propose an inverse optimization model to determine the objective weights for a general routing problem. Second, we conduct a detailed analysis of a case study that incorporates real weather information. Specifically, we evaluate the proposed approach using pre-generated routes (with given weights) and analyze the resulting weights and the generated routes. Third, we compare the performance of the inverse optimization and AHP methods using best practice routes provided by an experienced captain and route planner.

The remainder of the paper is structured as follows: Section 2 provides a description of maritime vessel routing problems and decision-making methods to establish the theoretical background. We also present a brief formulation of the safety issues, fuel consumption, and voyage duration, which are the key factors considered in this study. Section 3 outlines our proposed inverse optimization model, offering a detailed step-by-step description of the mathematical programming approach employed. To demonstrate the practical applicability of our model, Section 4 presents a case study that showcases its effectiveness. In Section 5, we present the results of our analysis using a set of pre-generated best practice routes. Finally, the discussion and conclusion are presented in Sections 6 and 7, respectively.

2 Background

2.1 Multi-criteria decision-making methods

There are various methods for estimating decision-making priorities to find the right balance between goals, including subjective and objective methods. Subjective decision-making involves evaluating different options based on personal judgments, values, and opinions, while objective decision-making involves analyzing options using real and measurable data. In multi-criteria vessel route planning, DMs evaluate different options based on various criteria, such as safety, fuel consumption, and voyage duration. However, this evaluation process can be subjective as DMs weigh the importance of each criterion based on their personal judgments, values, and preferences in focus groups, surveys and questionnaires.

Objective decision-making approaches in multi-criteria vessel route planning rely on verifiable and measurable data. DMs analyze various options using mathematical models and algorithms to determine the most suitable option based on predetermined criteria. Such approaches provide a reliable and transparent process for making decisions, but they depend on the quality of data and the expertise of DMs. AHP, entropy weighting, standard deviation, and statistical variance are objective decision-making approaches. We accordingly provide a comparison of objective and subjective methods to evaluate them in addressing the specific decision-making challenges in our context.

There exist subjective methods such as point allocation, ranking, and swing weighting in multi-criteria decision-making (Odu, 2019). In point allocation, DMs assign numerical values to each criterion based on their perceived importance, and then evaluate the criteria based on these weighted scores. In ranking, DMs order the criteria according to their preference and select the best criteria based on this ranking. The swing weighting method offers a systematic approach to decision-making. The process starts by choosing a set of objective weights and then identifying the objectives with the least weight. Next, the DMs select an objective that they believe has the potential to change its weight from the lowest to the highest. It is referred to as the reference objective and often being assigned a weight of 1 as the most desirable outcome. Afterward, the DMs assess other criteria by comparing their relative importance to the reference criterion, represented by values between 0 and 1. Through the process of normalization, the weights are appropriately adjusted to ensure fairness, ensuring that all criteria collectively sum up to 1. The reason behind this method is to assess the impact of the reference objective on the overall decision-making process. By iteratively adjusting the weight of the objective from lowest to highest, DMs can clearly see how changes in this weight significantly influence the final decision outcome.

In general, subjective methods offer various advantages, enhancing decision-making. They provide flexibility to incorporate qualitative factors, expert insights, and implicit knowledge that may be hard to quantify. This adaptability proves valuable for complex and multidimensional issues where objective data alone is not enough. Moreover, subjective methods allow for quicker decisions, as they do not rely on extensive data analysis or complex calculations. They allow for considering different viewpoints and values of stakeholders, encouraging involvement and alignment with organizational goals. However, subjective methods also have drawbacks. One concern is bias and inconsistency, where personal opinions and emotions can lead to unfair weight assignments. Lack of transparency makes it hard to justify decisions. Reproducibility is an issue as different DMs may assign different weights, affecting reliability. Calibrating weights through subjectivity can be challenging and impact decision quality. For instance, point allocation may lead to inconsistent and unreliable evaluations if DMs' weight assignments are influenced by personal biases. Similarly, ranking may not consider the trade-offs between different criteria, which may lead to suboptimal decisions. In swing weighting, the process of adjusting weights can be time-consuming and complex, and DMs may not have the necessary expertise to carry out this process effectively. Therefore, the subjective nature of these methods can limit their reliability and applicability in certain decision-making contexts.

AHP is a decision-making methodology used to simplify complex decisions by breaking them down into smaller, more manageable parts. AHP involves creating a hierarchical structure for the decision problem, where criteria are organized in a logical order. Each criterion is then systematically compared against others on a pairwise basis, establishing their relative importance and enabling DMs to assign appropriate weights to these criteria. Although the pairwise comparisons are essentially subjective to the DM's judgment, AHP is considered an objective method due to its incorporation of consistency checks into the decision-making process. The consistency ratio allows DMs to assess the reliability of their judgments when making pairwise comparisons. It helps in minimizing the influence of random or subjective biases, leading to more robust and dependable results. However, AHP is not entirely free from bias. One of the primary sources of bias lies in the inference of the DM's judgment during the pairwise comparison process. Since the evaluation of criteria and alternatives relies on the individual's perceptions and opinions, there is a possibility of inherent biases in the decision-making process. Such biases may arise due to personal preferences, experiences, or cognitive limitations, and they can impact the overall accuracy and reliability of the AHP outcomes. Furthermore, this method can be time-consuming and resource-intensive, which may restrict its applicability in certain contexts. As the number of objectives increases, the comparison between each pair becomes more complex, requiring expertise in the AHP method to recognize the complexity. Expertise in AHP is crucial to not only manage the increased number of comparisons but also to understand and interpret the growing complexities these additional objectives introduce in the decision-making process. In such cases, if additional objectives are added to the model, the entire process must be reapplied with even greater complexity, as the number of comparisons increases.

The entropy weighting system is a method rooted in data analysis, utilized in multi-criteria decision-making to objectively assign importance to distinct criteria (Zou et al., 2006). Based on the entropy principle from information theory, this technique measures the inherent variation or unpredictability of each criterion among a set of options. The process begins with data standardization, guaranteeing that all values fall within a consistent range. Within this standardized data set, a matrix is formed where every row denotes an alternative and each column represents a criterion. The values in this matrix reflect the standardized performance of alternatives based on the corresponding criteria. Subsequently, entropy for each criterion is evaluated based on how random or unclear it is among the considered alternatives. For each criterion, the level of diversification is determined by subtracting its entropy value from one. Finally, weights are allocated to the criteria according to their diversification levels, with criteria having lower entropy (meaning less randomness) receiving a higher weight. This is based on the idea that criteria with greater variation would be more critical compared to others. The standard deviation/statistical variance method uses the same matrix structure but concentrates on analyzing the standard deviation or statistical variance of the matrix's elements, offering a different perspective in assessing the importance of each criterion (Odu, 2019).

Despite their objective nature, the above methods (i.e., entropy, standard deviation, and statistical variance) have their limitations. First, they presume that the behavior of the data is directly linked to importance, which may not consistently align with real-world contexts. Additionally, the approach might not reflect the personal preferences of the DMs, potentially neglecting essential context-specific information. Moreover, since these methods heavily rely on the quality and completeness of the data being used, inaccurate or incomplete data can lead to misleading results.

2.2 Multi-criteria vessel routing problems

In challenging weather conditions, maritime vessels face a multitude of hazards that could compromise their stability, cause damage to cargo and equipment, or pose significant risks to crew and passengers. The severity of these risks varies considerably across different types of ships, influenced by a range of factors including the vessel's stability characteristics, hull design, size, and operational speed. Consequently, when planning a ship's route, it is essential to consider a variety of safety concerns. The safety of maritime vessels encompasses a wide spectrum of considerations, adhering to standards and regulations established by international bodies such as the IMO and seakeeping performance.

Various key elements impact maritime safety, fuel efficiency, and voyage duration, including meteorological conditions like wind speed and direction, current speed and direction, as well as sea states described by wave height, direction, and period. For example, when facing headwinds, a ship maintaining the same engine output as in calm conditions will experience a reduction in velocity. To maintain the same speed, the vessel would need to increase engine output, which in turn would increase fuel consumption. Large waves impacting the ship's side can significantly affect its stability.

The IMO (2007) guidelines detail safety regulations that address phenomena such as surf-riding and broaching-to, which occur under specific conditions of wave and ship speed, potentially causing the vessel to severely heel or suddenly change direction. Other hazards include successive high-wave encounters that can lead to synchronous or parametric rolling motions, each associated with significant risks of capsizing due to instability. These phenomena are influenced by the relationship between wave characteristics and the ship's design and operational parameters, describing zones of safety and danger based on vessel characteristics and weather data. When these safety aspects are combined, they represent non-dangerous and dangerous zones that are limited by vessel and weather data (Sharif et al., 2024). In the context of optimization, the terms “dangerous zone” and “non-dangerous zone” refer to the hard and soft constraints, respectively.

Seakeeping performance, which includes how well a ship can maintain its course and speed in rough waters, is a critical component of vessel safety. Seakeeping analysis involves the use of mathematical models to simulate the interaction between ships and the fluid dynamics of the sea (Pennino, 2020). This analysis categorizes ship movements into six basic types: three linear (surge, sway, and heave) and three rotational (roll, pitch, and yaw), collectively known as seakeeping characteristics. Engineers use these models to predict how ships will respond to external forces such as waves, wind, and currents, considering factors like hull shape, weight distribution, and motion characteristics. This understanding is crucial for designing vessels that can safely navigate through adverse conditions.

The speed of a ship plays a pivotal role in navigating safety challenges associated with maritime motions. Adverse weather introduces additional resistance from waves, wind, and ship movements, slowing the vessel's progress and increasing fuel consumption. The interaction between the ship's hull and the water generates added resistance, which depends on various factors including wave height, ship speed and orientation, hull design, and overall sea conditions. Overcoming this added resistance is crucial for maintaining efficient navigation and minimizing the impact of adverse weather on maritime operations.

The following section presents a background review of prior studies within the field of maritime research that have delved into safety considerations within the context of multi-objective routing problems.

The Multi-Criteria Evolutionary Weather Routing Algorithm (MEWRA) stands as a sophisticated navigational tool, designed to determine the most efficient maritime path while balancing the often-conflicting objective of safety, time, and fuel consumption (Szłapczynska and Smierzchalski, 2009). Rooted in the principles of Multi-Objective Evolutionary Algorithms, MEWRA strives to provide a set of near-optimal solutions that best satisfy these diverse criteria. It initiates this process by creating a base population of potential routes, each representing a unique solution. These routes undergo iterative refinement through crossover and mutation processes for introducing variability and ensuring continuous improvement in search outcomes. The algorithm methodically evolves, continuously seeking improved solutions until it reaches a specified conclusion point, which might be determined by generation count, elapsed time, or achievement of a satisfactory solution set.

Maritime navigation has significantly advanced with the use of sophisticated algorithms that optimize vessel routing by considering key factors like time, fuel consumption, and safety. A notable contribution in this field is the MEWRA developed by Krata and Szlapczynska (2012), which integrates a safety index into the route planning process. This index, designed in alignment with the IMO (2007) guidelines, distinguishes between dangerous and non-dangerous zones by evaluating potential hazards like surf-riding, broaching-to, and successive high-wave attacks. In earlier work, Szlapczynska (2015) underscores the importance of dynamic and static constraints in route optimization, where static constraints include fixed geographical obstacles, and dynamic constraints adjust based on varying weather conditions.

Further expanding on this concept, Fabbri and Vicen-Bueno (2019) proposed a multi-criteria vessel routing problem that incorporates a detailed assessment of ship navigation resistance and safety risks associated with IMO (2007) dangerous zones, utilizing Martins' labeling algorithm (Martins, 1984) for solution finding. This algorithm, by prioritizing paths based on a set of weighted objectives, facilitates the exploration of Pareto optimal solutions, offering a multifaceted understanding of trade-offs between various objectives. Similarly, Veneti et al. (2017) introduced a labeling algorithm inspired by Martins' algorithm, aiming to minimize fuel consumption and safety risks related to IMO (2007) within a framework that also considers voyage time constraints. This method, by efficiently managing the balance between safety and operational efficiency, demonstrates the evolving complexity of maritime route planning algorithms. The work by Krata and Szlapczynska (2018) further exemplifies the trend towards multi-objective optimization in ship routing, addressing the need to balance fuel consumption, voyage duration, and safety. Their methodology, estimating Pareto frontier through the MEWRA approach, introduces a nuanced consideration of stability factors, such as the vessel's metacentric height (GM), into the optimization process. This inclusion of stability metrics highlights the importance of vessel design characteristics in determining safe and efficient navigational paths.

The research attention towards seakeeping performance is also relevant. Zaccone et al. (2018) focused on minimizing fuel consumption while ensuring compliance with safety constraints related to the vessel's motion responses to sea conditions. Their approach, utilizing the spectral moment of order zero and the response amplitude operator (RAO), showcases the application of physical and mathematical principles in evaluating the safety and efficiency of maritime routes. This emphasis on the ship's physical response to environmental conditions underlines the critical interplay between vessel design, sea state, and navigational safety. Padhy et al. (2008) explored RAO using Dijkstra's algorithm to minimize travel time by considering ship speed. They suggested that reducing speed improves safety through enhanced seakeeping. Safety, as a hard constraint, was evaluated via RAO analysis for different hulls, speeds, and headings, influencing speed adjustments in route optimization.

Innovative methodologies continue to emerge, as demonstrated by Pennino et al. (2020), who employed Dijkstra's algorithm to optimize routing based on the Seakeeping Performance Index (SPI), a comprehensive measure of a vessel's operational efficiency and safety. Vettor et al. (2020) developed a multi-objective metaheuristic approach that used probabilistic methods to evaluate seakeeping criteria and weather-related risks. The emphasis on measurable indices highlights the continuous work to enhance route optimization criteria, making sure that decisions are based on objective evaluations of how vessels perform in different sea conditions.

The above papers focus on integrating safety, fuel consumption, and route time in maritime routing, incorporating safety as either a soft or hard constraint according to IMO standards and seakeeping criteria. This underscores the emphasis on adhering to global standards in maritime research. Yet, the simultaneous consideration of an extensive number of safety issues, along with fuel consumption and route time, is still unexplored. There is a pressing need to encompass all safety dimensions in vessel routing to ensure comprehensive safety for cargo, vessel integrity, and crew/passengers. Incorporating these safety objectives introduces greater complexity in setting objective weights and optimizing routes, highlighting the essential need for advanced decision support tools for complex multi-objective maritime routing.

2.3 The formulation of the safety, fuel consumption, and time criteria

2.3.1 Safety issues calculation for maritime vessels

In this study, we focus on five principal objectives: three safety-related criteria, as well as fuel consumption, and voyage duration. The research highlights three pivotal safety concerns: dynamic stability, the probability of green water on deck, and bow slamming. These concerns are integrated into the optimization process, functioning as both hard and soft constraints. The safety hard constraints are non-negotiable boundaries in the solution space, ensuring that solutions remain within defined safety thresholds. Conversely, the soft constraints are integrated into the objective function, which is aimed to be minimized throughout the optimization process.

Theoretically, bow slamming and green water probabilities are computed based on an exponential function related to SHIPX Vessel Responses (VERES) ship motions, while dynamic stability criteria are determined by computing the absorbed energy affected by wind speed and direction and comparing it to the vessel's metacentric height and displacement. The following sections offer brief explanations and models of the safety criteria. For a comprehensive analysis and detailed formulation of this probability, readers are referred to the extensive discussion and modeling provided in Sharif et al. (2024).

Table 1 outlines the parameters associated with vessel characteristics, and the environmental data related to weather conditions, which are used to formulate the three safety criteria. In Figure 1, the primary dimensions of a cargo vessel, namely its length, draft, and freeboard, are depicted. These measurements play a crucial role in assessing the stability and safety of a vessel in different sea conditions.

Table 1. The input in the calculation for safety criteria.

Symbol	Description
Vessel information	
GM	Metacentric height
∇	Mass displacement
L_{OA}	Length
d	Draft
fb	Freeboard
Weather data	
v_{wt}	Wind speed
Ψ_{wt}	Wind direction
H_S	Significant wave height
T_1	Mean wave period of irregular waves

Draft is another important metric, representing the vertical distance from the waterline to the ship's lowest hull point. This dimension is key in determining the depth to which the ship is submerged in the water, especially when fully loaded and when vessels need to pass under infrastructure such as bridges or powerlines. In such cases, draft is sometimes distinguished by water draft, indicating the depth below the waterline, and air draft, indicating the height above the waterline to ensure clearance. In contrast, the freeboard measures the vertical span from the waterline to the ship's main deck, indicating the margin of safety to prevent water from entering the deck.

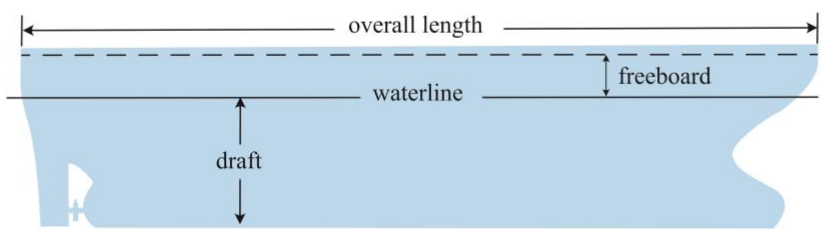


Figure 1. Waterline, overall length, draft, and freeboard of a vessel.

2.3.1.1 Dynamic stability

The dynamic stability of a ship is the vessel's potential to absorb the external forces to a certain angle to prevent capsizing. When a ship is tilted, the center of gravity (G) and the center of buoyancy (B) form vertical lines that do not align, creating a horizontal gap between them. This gap defines the righting arm (GZ), which is crucial for understanding the ship's ability to return to an upright position. The relationship between this righting arm and the ship's heel angle is graphically represented by the GZ curve, a critical graph that highlights the ship's stability. A positive righting arm signals the ship's effort to restore itself, while a negative one points to potential instability, raising the risk of the ship capsizing. This study specifically focuses on the influence of wind as the primary external force that causes the ship to tilt. The dynamic stability criterion is formulated based on the magnitude force of the wind that causes the ship to tilt (*a*) and the ability of the ship to resist the imposed pressure (*b*). Figure 2 provides a graphical representation of these forces, indicating the wind's tilting force (area *a*) against the ship's righting moment (area *b*), essential for understanding the conditions for dynamic stability as per DNV GL (2019). Therefore, the vessel would be in stable equilibrium when *b* is bigger than *a* (DNV GL, 2019). For a more comprehensive understanding, detailed information on the computational process can be found in Sharif et al. (2024).

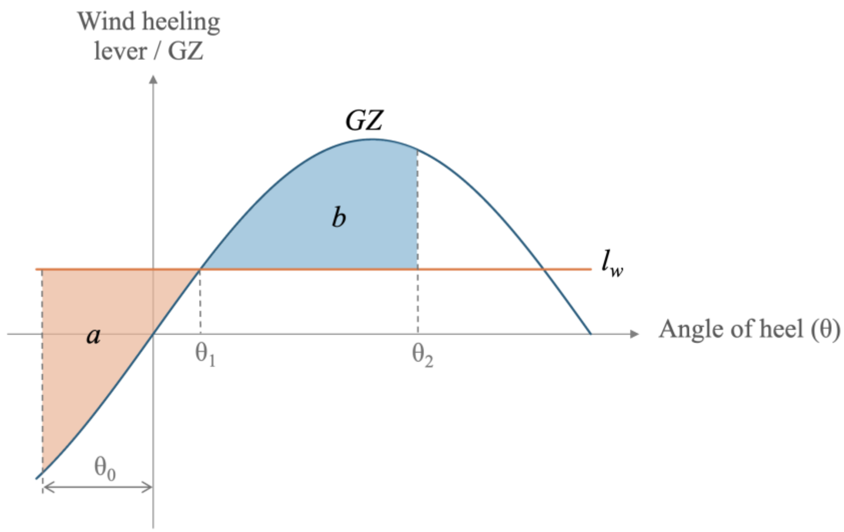


Figure 2. Illustration of wind force magnitude (l_w) that causes ship to tilt (represented by area *a*) and the ship's righting moment that resists tilting (represented by area *b*), according to different heeling angles (θ_0, θ_1, θ_2), used for assessing dynamic stability.

2.3.1.2 Green water on deck

In adverse weather, ship motions can become excessively large, which causes the water to flow onto a vessel's deck. This phenomenon threatens the ship's structure (i.e. deck plating, hatched), cargo and crew. The assessment of this risk involves considering factors like the vessel's freeboard and the variance in wave elevation. To ensure safety, the risk associated with this phenomenon should not exceed a specific threshold. A detailed mathematical formulation of this probability, based on NORDFORSKs' assessment and Nielsen’s (1987) approach, can be found in Sharif et al. (2024). This provides a thorough analysis of the green water risk and its implications for ship safety.

2.3.1.3 Bow slamming

Slamming is the action of high waves raising the bow of a vessel on the sea surface that causes physical damage to the ship's crew and system. Investigating a vessel's motions subjected to waves is a crucial problem that can be obtained through the ship's design and weather data. The probability of bow slamming, using Nielsen's method (1987) and NORDFORSKs' assessment, depends on factors such as the ship's draft, threshold velocity, variations in wave elevation, and velocity variances. In order to maintain optimal safety standards, it is crucial to ensure that the risk of bow slamming remains within a specified safety threshold. A thorough analysis on this topic can be found in Sharif et al. (2024).

2.3.2 Voyage duration and fuel consumption

The role of speed is essential in determining the voyage duration. There are two primary types of speed: speed through water (STW) and speed over ground (SOG). On the one hand, STW indicates the ship's velocity relative to the surrounding water, playing a critical role in assessing engine and hull performance in water. It is influenced by elements such as design and condition of the ship, the effectiveness of its propulsion system, and the density of the water. On the other hand, when estimating the actual time taken for a voyage, SOG becomes more significant. It represents the ship's speed in relation to the Earth's surface, quantifying its rate of movement between geographical points, while considering the impact of water currents (Yang et al., 2020). This distinction is crucial because strong currents can cause SOG to differ from STW. Ultimately, voyage duration is calculated by dividing the total distance traveled by SOG, providing a more precise measure.

In maritime navigation, managing fuel consumption is also crucial due to its significant impact on both financial and environmental aspects. It constitutes a major part of a ship's operating costs and plays a significant role in the emission of greenhouse gases. Fuel consumption is determined by various factors, including vessel design, engine efficiency, revolutions per minute, and a range of environmental factors like wind, sea waves, and ocean currents. Each of these components individually and collectively influences how much fuel a ship consumes during its journey. Incorporating these varied factors into a predictive model, like a regression model employed in this research, enables more precise and reliable estimations of fuel consumption. For more detailed information, we refer to Hajli et al. (2023). Moreover, Yan et al. (2021a) conducted a thorough review and comparative analysis of ship fuel consumption prediction models.

3 Model and methodology

Inverse optimization is an effective method for identifying appropriate objective weights that reflect the real-world priorities in multi-objective problems. Unlike traditional optimization methods, where the process starts by setting specific objectives and then seeking the most effective solutions to optimize those objectives, inverse optimization follows a unique approach. It starts with observed solutions and works backward to infer the objectives or preferences that led to them.

Weight estimation through inverse optimization can potentially mitigate the issue of sensitivity to weights in the weighted sum method. By inferring weights from actual decision-making data rather than relying on subjective assignment, it introduces a data-driven approach. This means that the weights are grounded in real-world outcomes and practices, which may be more stable and representative of actual priorities and trade-offs made in practice.

The approach adheres to the optimality conditions introduced by Kuhn and Tucker (1951), which are known as Karush-Kuhn-Tucker (KKT) conditions. These conditions are a set of requirements that, when satisfied, provide necessary conditions for a point to be a local optimum in a nonlinear optimization problem with certain constraints. The KKT conditions are crucial to ensure that the inferred parameters formulate an optimization problem where the provided solutions are optimal. They guarantee that solutions not only achieve optimality but also maintain feasibility, all while adhering to the constraints of the derived optimization problem.

The KKT conditions encompass three key elements: *primal feasibility*, *dual feasibility*, and *complementary slackness*. *Primal feasibility* is a straightforward but fundamental requirement of KKT that insists all solutions reside strictly within the set boundaries of the problem, adhering to all constraints. *Dual feasibility* identifies a state of balance where the direction and magnitude of the objective function's gradient is a non-negative linear combination of the gradients from the active constraints. It also ensures that Lagrange multipliers tied to the constraints are non-negative, reflecting the concept that one cannot obtain a better solution by violating the constraints. *Complementary slackness* creates a direct link between the constraints and their corresponding multipliers: If a constraint is not satisfied with equality, its multiplier must be zero.

Inverse optimization has a vast range of applications in different areas, such as project portfolios (Roland et al., 2016) and transportation (Flisberg et al., 2012; Rönnqvist et al., 2017). Flisberg et al. (2012) applied the Dijkstra algorithm to identify the minimum cost (MC) route based on different road characteristics. The allocation of suitable weights to each of these characteristics posed a significant challenge. To address this, the researchers applied the concept of inverse optimization, which facilitated the determination of weights according to drivers' preferences expressed through a set of best practice routes. By employing this concept, the primary objective of this study was to maximize the number of MC's optimal routes that align with the best practice routes. The model was formulated as a large-scale Mixed Integer Programming (MIP) model and was solved using column and row generation techniques. This approach led to the development of optimized routes that were better aligned with drivers' preferences compared to other weight estimation methods. This study not only enhanced the effectiveness of transportation planning but also ensured the optimization tool would effectively reflect drivers' preferences. Chan et al. (2023) conducted a comprehensive review encompassing the theory and applications of inverse optimization approach, shedding light on its diverse applications and underlying principles.

3.1 Multi-criteria minimum cost flow problem

The route planning problem involves identifying the most efficient path by solving a minimum cost flow problem, which considers the weight of the objectives. This approach is crucial due to the multi-dimensional nature of the objectives involved. Unlike the shortest path problem which identifies a single optimal route, the minimum cost flow framework enables the simultaneous analysis of multiple best practice routes. Our scenario involves not just a single vessel traveling between two points, but potentially multiple vessels that must be coordinated across a network. This network is constructed through a set of nodes and arcs. The nodes represent specific waypoints or significant maritime locations such as ports, anchorage areas, or navigational landmarks. These nodes are not static but rather dynamic, as they incorporate information related to travel time based on factors such as distance and vessel speed. This route time information allows for the consideration of varying time intervals in the network, ensuring that routing decisions account for both spatial and temporal factors. Finally, the arcs act as connections between these nodes.

The objective function of the minimum cost flow problem aims to minimize the total weighted objective functions. In this model, a tree structure of the best practice routes is essential as it ensures the unique definition of each path from the origin to its destination. This feature is crucial for effectively representing the best practice routes. Such a structure is achieved by enforcing either a single origin or a single destination node (a more general case is discussed in Section 6).

Below, we provide definitions for the variables and parameters in the model.

Sets

B	set of arcs
N	set of nodes
S	set of safety issues
M	set of objectives

Parameters

b_k	node balance $(k \in B)$
t_{ij}	voyage duration from node *i* to node *j* $\left((i,j) \in B\right)$
f_{ij}	fuel consumption from node *i* to node *j* $\left((i,j) \in B\right)$
s_{hij}	quantified safety indices for each arc $\left((i,j) \in B_, h \in S\right)$
w_m	objective weights $(m \in M)$
c_{ij}	arc cost $\left((i,j) \in B\right)$

Variables

x_{ij}	flow (number of vessels traveling) between node *i* and node *j* $\left((i,j) \in B\right)$

The parameter b_k denotes the net change in the number of vessels at a node, reflecting the balance of incoming and outgoing vessel movements. This value varies according to the position of the node under consideration. Specifically, it takes the value of -1 at the origin node, +1 at the destination node, and 0 at an intermediate or interconnecting node. In a network configuration with one origin and n distinct destinations, the origin node will have $b_k = -n$, reflecting the departure of vessels to multiple destinations. Conversely, each destination node will have $b_k = +1$. The opposite holds true for a network with multiple origins and a single destination: b_k becomes $+n$ at the destination and -1 at each origin.

Given these notations, Equation (1) represents the cost function proposed in this study. This function is formulated as a weighted sum of objectives, such that the total weight is one.

$$c_{ij} = w_1 t_{ij} + w_2 f_{ij} + \sum_{h \in S} w_h s_{hij} \qquad (1)$$

Within the context of the problem, the index h belongs to the set of safety issues (S) in s_{hij} $\big((i,j) \in B, h \in S\big)$.

The minimum cost flow problem is given as the following:

Model P:

$$\min \sum_{(i,j)\in B} c_{ij} x_{ij} \tag{2}$$

subject to

$$\sum_{i:(i,k)\in B} x_{ik} - \sum_{j:(k,j)\in B} x_{kj} = b_k \qquad \forall k \in N \tag{3}$$

$$x_{ij} \geq 0 \qquad \forall (i,j) \in B \tag{4}$$

The objective function of model P (2) minimizes the total transportation cost. Constraints (3) correspond to the node balance and Inequalities (4) relate to the non-negativity requirements on the flow variables.

The presence of multiple objectives, combined with corresponding weights, necessitates the implementation of a normalization method to ensure comparability among the objective function values. To address this, a linear normalization technique is employed for each objective value, aiming to align them within comparable intervals. The process involves subtracting the minimum value from the original value and subsequently dividing this result by the amplitude, which represents the difference between the maximum and minimum values. This approach ensures uniform standardization by rescaling the original values, regardless of their initial units or scales, to uniformly fall within a standardized range of 0 to 1. The formula for calculating the normalized value is as follows:

$$\text{Normalized Value} = \frac{\text{Original Value} - \text{Minimum Value}}{\text{Maximum Value} - \text{Minimum Value}} \tag{5}$$

For instance, consider an original value of 0.04 for green water probability, within a data range from 0.01 to 0.07. According to the normalization process, the normalized value is calculated to be 0.5.

3.2 Dual programming

Denoting by y_k $(k \in N)$ the multiplier for each of the node balance constraints in model P, the dual problem is defined as follows:

Dual:

$$\max \sum_{k \in N} b_k y_k \tag{6}$$

subject to

$$y_i - y_j \leq c_{ij} \qquad \forall (i,j) \in B \tag{7}$$

The objective function of Dual problem (6) contains the dual variables of model P multiplied by the right-hand side of the node balance constraints. Constraints (7) ensure that the difference between the dual variables at nodes i and j does not exceed c_{ij}, $(i,j) \in B$, which represents the allowable increase in the cost when traveling from node i to node j.

Our methodology incorporates the principles of both strong and weak duality. Weak duality dictates that the optimal objective function value of the dual problem is never smaller than the optimal objective function value of primal model for any feasible solution (x_{ij}, y_k).

Given any feasible solution $\left(x_{ij}, y_k\right)$, $\left((i,j) \in B, k \in N\right)$ to model P and Dual problem, respectively, the weak duality is as follows:

$$\sum_{k \in N} b_k y_k \leq \sum_{(i,j) \in B} c_{ij} x_{ij} \tag{8}$$

Strong duality asserts that optimal solutions (x_{ij}^*,y_k^*) to the primal and dual problems yield the same objective function value, provided that bounded primal and dual solutions exist. It is represented as follows:

$$\sum_{k \in N} b_k y_k^* = \sum_{(i,j) \in B} c_{ij} x_{ij}^* \qquad (9)$$

3.3 KKT optimality conditions

In Linear Programming (LP), KKT refers to the optimality conditions involving both primal and dual feasibility (Ghojogh et al., 2021). These conditions are crucial in establishing whether a feasible solution is optimal. Consider a given general LP problem where *c* is the cost coefficient vector, *x* is the decision variable vector, *A* is the coefficient matrix for constraints, and *b* represents the right-hand side values for constraints:

$$\min c^T x \qquad (10)$$

subject to

$$Ax = b \qquad (11)$$

$$x \geq 0 \qquad (12)$$

Denoting by y the dual variables or Lagrangian multipliers, the KKT optimality conditions to the problem are:

Primal feasibility: $Ax = b\ , x \geq 0$ (13)

Dual feasibility: $A^T y = c\ , y \in \mathbb{R}$ (14)

Complementary slackness: $y^T(b - Ax) = 0$ (15)

The best practice routes are defined by the decision variables $x_{ij}, (i, j) \in B$, to the model P. Those variables are assigned the value of flow for the arcs that constitute the route, while all others receive a zero. The foundation of these best practice routes lies in expert insights (factors such as safety, fuel efficiency, and route times), considering specific weather data.

Following the same logic, and the definition of best practice routes, where $\bar{x}_{ij}, \big((i, j) \in B\big)$ is the parameter representing the best practice arcs, we can define the KKT conditions for our problem. The value of flow variables on each arc (either $\bar{x}_{ij} > 0$, or $\bar{x}_{ij} = 0$) plays an important role in influencing the complementary slackness condition. If the arc has a positive flow value ($\bar{x}_{ij} > 0$), the reduced cost on that arc should be zero. Conversely, for arcs with no flow ($\bar{x}_{ij} = 0$), the reduced cost should be positive. Hence the final KKT condition for our problem can be formulated as:

Stationarity and dual feasibility:

$$y_i - y_j = c_{ij} \qquad \forall (i,j) \in B, x_{ij} > 0. \qquad (16)$$

$$y_i - y_j \leq c_{ij} \qquad \forall (i,j) \in B, \bar{x}_{ij} = 0 \qquad (17)$$

Model *P* feasibility and complementary slackness:

$$y_k \left(b_k - \sum_{i:(i,k) \in B} \bar{x}_{ik} + \sum_{j:(k,j) \in B} \bar{x}_{kj} \right) = 0 \qquad \forall k \in N \qquad (18)$$

$$x_{ij} \geq 0 \qquad \forall (i,j) \in B \qquad (19)$$

The best practice routes represent a feasible solution of model *P*, thereby adhering to the corresponding KKT feasibility conditions.

3.4 Inverse programming

Our approach uses an inverse problem formulation that incorporates KKT optimality conditions, a weighted objective function description, and best practice route information. The inverse model is formulated as follows:

Model I:

$$\max \sum_{k \in N} b_k y_k \tag{20}$$

subject to

$$y_i - y_j = w_1 t_{ij} + w_2 f_{ij} + \sum_{h \in S} w_h s_{hij} \qquad \forall (i,j) \in B, \bar{x}_{ij} > 0 \tag{21}$$

$$y_i - y_j \leq w_1 t_{ij} + w_2 f_{ij} + \sum_{h \in S} w_h s_{hij} \qquad \forall (i,j) \in B, \bar{x}_{ij} = 0 \tag{22}$$

$$\sum_{k \in N} b_k y_k = \sum_{\forall (i,j) \in B} (w_1 t_{ij} + w_2 f_{ij} + \sum_{h \in S} w_h s_{hij}) \, \bar{x}_{ij} \tag{23}$$

$$w_1 + w_2 + \sum_{h \in S} w_h = 1 \tag{24}$$

$$w_m \geq 0 \qquad \forall m \in M \tag{25}$$

The objective function (20) is based on dual programming, with Constraints (21) and (22) corresponding to the KKT optimality conditions. Equation (23) imposes a strong duality between model P and Dual problem. The sum of the objective weights is set to be one by (24), and these weights are required to be positive by (25).

4 Case study

The study described in this paper was carried out in close collaboration with True North Marine (TNM), a Canadian company. TNM is a consultancy firm dedicated to assisting bulk vessel operators in proposing efficient route plans balancing cost and safety over the entire voyage. The vessel used in this case study is the bulk carrier DOUBLE DIAMOND, whose the characteristics are provided in Table 2. In the specific case being discussed here, the speed (SOG) of the vessel is considered to be a fixed parameter that remains constant throughout the optimization. Moreover, we consider a case study with one origin and multiple destinations. However, it should be noted that even in scenarios involving multiple origins with a single destination, or multiple origins and destinations, the logic for the node balance parameters remains valid. It is adapted appropriately to account for the specific characteristics of each setup.

To study the efficiency of the inverse optimization model we used manually set weights for this setup to generate the best practice routes. We then analyzed if it was possible to re-establish the assumed weights and routes by only using information about the best practice routes. This not only strengthens our capacity to evaluate and contrast these initial weights with those derived from the inverse model, but also aids in the analysis of route cost.

The best practice routes were generated starting from one origin (Florida, USA) to eleven destination points (from Northwest Africa to the United Kingdom) crossing the North Atlantic Ocean. The definition of the network included 2,051 nodes and 5,851 arcs (Figure 3).

Table 2. The characteristics of the vessel used in the case study.

Basic Vessel Dimension	Value	Unit
Length between perpendicular	177.00	m
Length between waterline	173.40	m
Breadth	28.60	m
Load line	Summer	-
Draft	10.03	m
Main engine power	7,466.81	HP
Mass displacement	41,484	t
Ship depth	16.00	m
Freeboard	4.39	m
Center of gravity	3.79	m

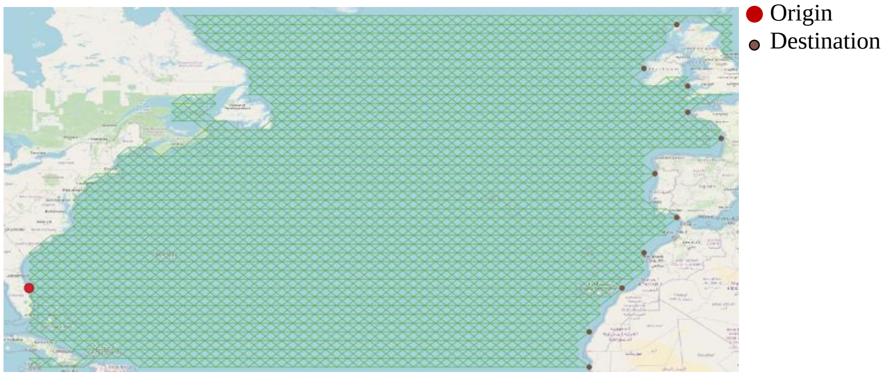


Figure 3. The network of the case study.

The weather data is considered to be static over the planning period. The main purpose is to enable an easier visual inspection of the results in the network. We used weather data for wind and wave direction and magnitude. The plotted weather data on the given network is illustrated in Figure 4. In the context of wave height, wave period, and wind speed, darker colors correspond to higher values, with the color range varying from white to red. Specifically, white arcs represent the lower values, while the intensity of red color deepens with increasing value magnitude.

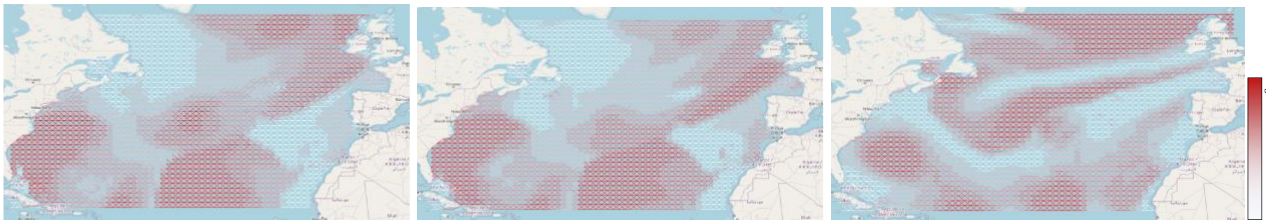


Figure 4. The data patterns of wave height (left), wave period (middle), and wind speed (right) on the network.

The computed values for the objectives are subject to variations from one objective to another. Specifically, the results obtained from the employed approaches for determining the safety criteria were constrained to the [0,1] interval, while fuel consumption and time exhibited distinct values. By applying linear scalarization to all objectives, a standardized and comparable scale was established. Consequently, all criteria were scaled within the interval of [0,1]. It should be noted that to adhere to safety standards, any solution exceeding the specified safety thresholds is implicitly removed from the network, as it is deemed infeasible.

The computed safety indices based on the abovementioned formulation are illustrated in Figure 5 using the same legend of color range. As expected, the probabilities of bow slamming and green water follow wave height and wave period, respectively. The same synergy was observed between dynamic stability criteria and wind speed. The behavior of the fuel consumption and voyage duration is also provided in Figure 5.

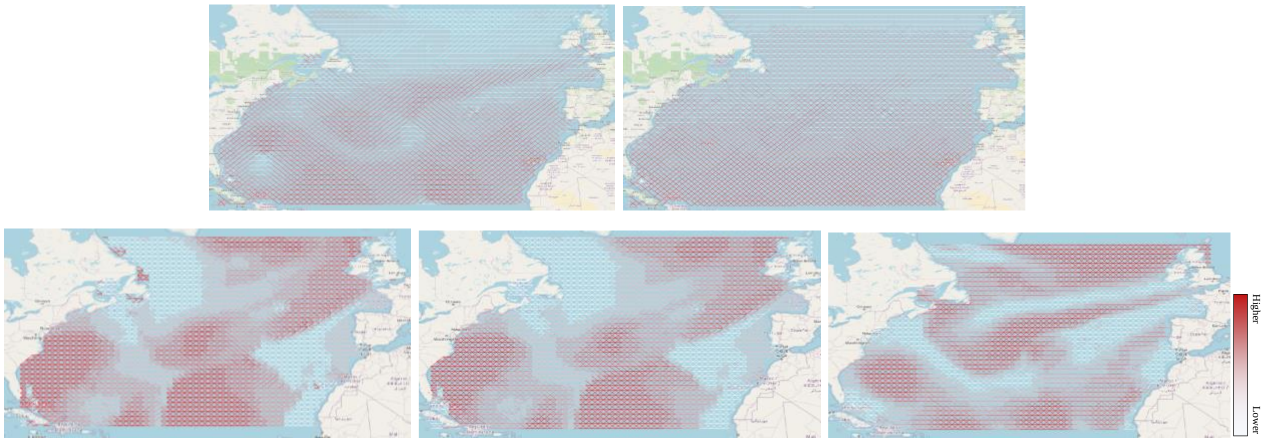


Figure 5. Plotted computed values on the network; fuel consumption (top left), time (top right), bow slamming (bottom left), green water on deck (bottom center), and dynamic stability (bottom right).

In order to evaluate our proposed approach, we utilized a set of objective weights to generate best practice routes, which served as the optimal solutions to the minimum cost flow problem (model *P*). Using this approach, we can avoid inconsistencies that arise from manually collecting best practice routes. Subsequently, the inverse model (model *I*) was employed to derive the objective weights associated with the best practice routes. These weights were then used to generate a new set of routes through model *P*, which were compared with the original best practice routes.

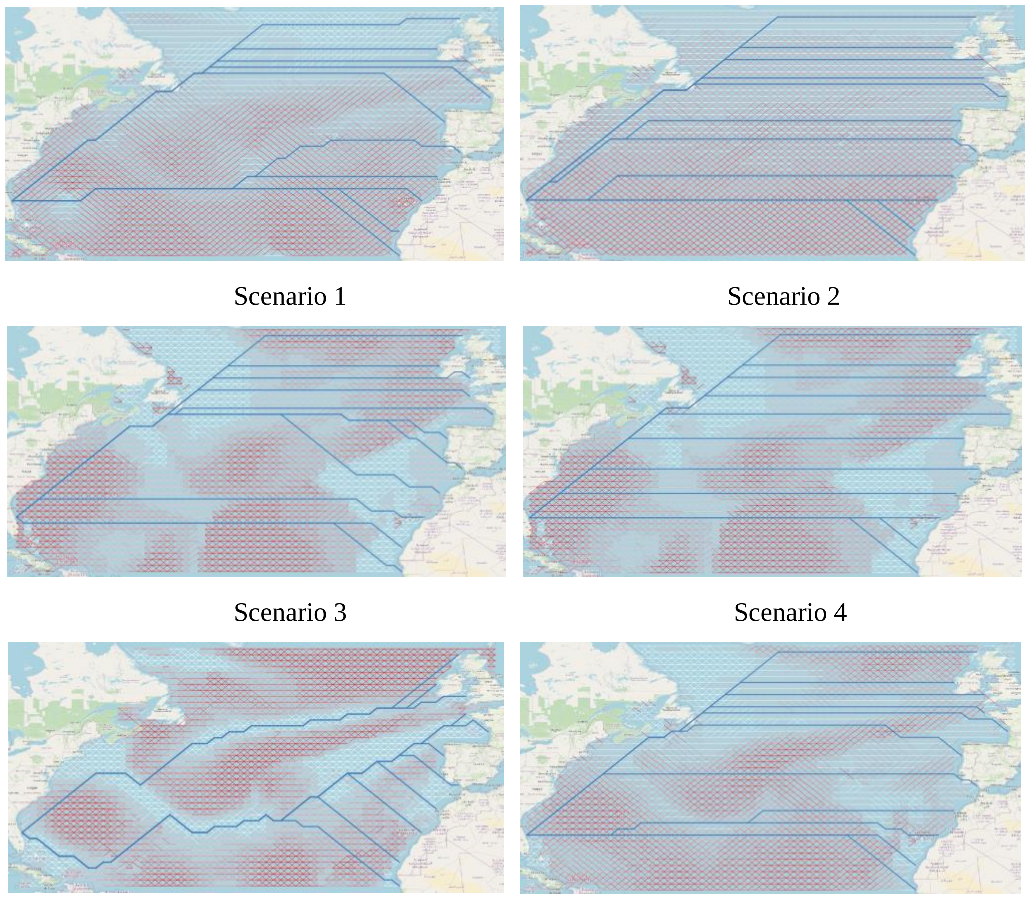
To determine the best practice routes, we conducted six scenarios with weight settings as shown in Table 3. These weights produced different routings illustrated in Figure 6. Scenarios 1-5 are equivalent to obtaining the optimal solution of the model *P* with respect to only one criterion (single objective function). In these setups, the impact of other objective values was assumed to be zero. For instance, if the objective was to minimize fuel consumption, a weight of one was assigned to this objective while the weights of other objectives were set to zero. In scenario 6, we generated the best practice routes by assigning equal weights to each objective at the same time. This phase aimed to demonstrate the efficiency of the developed approach in identifying the objective weights that led to the best practice routes with equal impact for all objectives.

To construct the interconnected network, we simultaneously added all objective values for each arc. The background network for each scenario was constructed by multiplying the associated objective value for each arc by the considered weight setting.

After deriving the objective weights through the inverse model, they can be incorporated into the minimum cost flow problem. This incorporation enables us to generate optimized routes that demonstrate the effectiveness of the derived weights. Subsequently, a comprehensive evaluation can be conducted between these newly generated routes and the established best practice routes. This evaluation can be visualized to evaluate the similarity between the generated and best practice routes. Additionally, a comparison between the initial and obtained weights can be achieved. Furthermore, given that the cost function considers the trade-off between various objective functions, the overall transportation cost of the routes can be analyzed before and after optimization to determine any cost savings resulting from the approach.

Table 3. Weight settings considered in each scenario.

Scenario	Route objective value	Route monetary value	Objective weight				
			Time	Fuel	Bow slamming	Green water	Dynamic stability
1	2,446,860	$3,817,330	1	0	0	0	0
2	2,289,920	$3,734,850	0	1	0	0	0
3	6,313,910	$3,874,220	0	0	1	0	0
4	7,208,200	$3,850,660	0	0	0	1	0
5	394,010	$4,586,970	0	0	0	0	1
6	3,847,550	$3,786,090	0.2	0.2	0.2	0.2	0.2



Scenario 1 | Scenario 2

Scenario 3 | Scenario 4

Scenario 5 | Scenario 6

Figure 6. The generated artificial best practice routes in the case study.

We also introduce another test case to conduct a comparative analysis between the AHP and our proposed approach for determining objective weights in multi-criteria vessel route problem. By undertaking this comparison, we aim to evaluate the efficiency of our approach in tackling multi-criteria problems and highlight any potential advantages or drawbacks when compared to the traditional AHP method. To achieve this objective, we have incorporated the hierarchical structure of the AHP method (Figure 7) that facilitates pairwise comparisons among all the objectives involved.

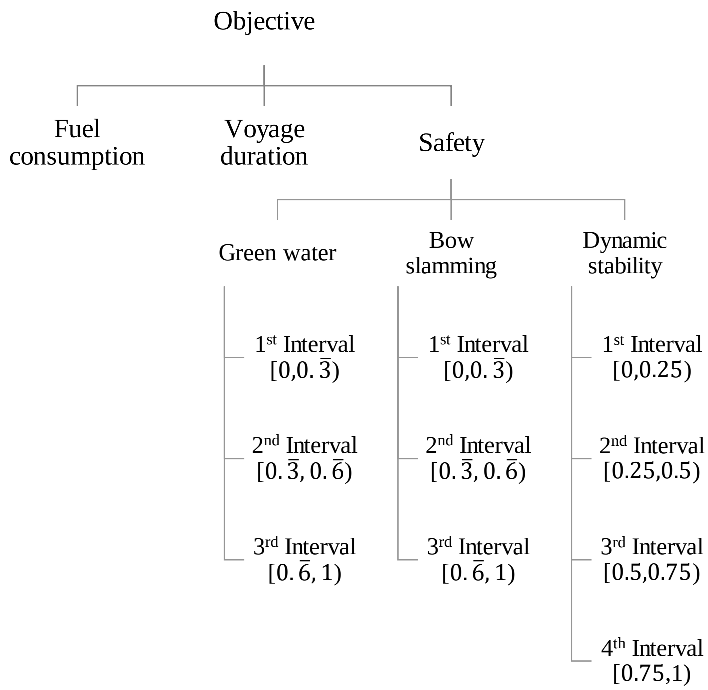


Figure 7. The hierarchy structure of AHP method used in this study.

It is noteworthy that given the non-linear nature of safety criteria, each was discretized into separate intervals. Specifically, the dynamic stability criterion was discretized into four intervals, while both the green water and bow slamming criteria were discretized into three intervals. Handling non-linear parameters directly in decision-making processes can pose challenges. Optimization problems encounter difficulties when dealing with non-linear parameters. In this regard, the weighting process becomes more complicated. Nonlinear interactions can alter how weights influence the solution, making it difficult to adjust the weights to align with DM preferences. Additionally, the solution becomes highly sensitive to weight changes, where minor adjustments may cause significant shifts in the outcome. However, by utilizing discretization, we can simplify the problem and make it more suitable for optimization methods. To overcome these challenges, we discretized the safety parameters by dividing them into separate intervals. This allowed us to control their nonlinear behavior and effectively integrate them into the mathematical model, ensuring comparability with the AHP method. By breaking down the parameters into intervals, we simplified their impact on the decision.

To facilitate the analysis of different safety issues, we segmented the data range into intervals of equal length. Specifically, for green water and bow slamming, we divided the range into three intervals, while for the dynamic stability, we used four intervals of equal length. The assignment of priorities to each interval was based on expert knowledge and their analysis. It is important to highlight that as the safety parameter increases, the nonlinearity also increases, leading to priorities being determined in ascending order.

The comparisons utilized a numerical scale that ranged from 1 to 9, with 1 indicating that the two elements possessed equal importance, and 9 indicating that one element was significantly more important than the other. The first stage indicated the upper-level objectives encompassing fuel consumption, time, and safety, as depicted in Table 4. The subsequent stage entailed evaluating and comparing the secondary-level safety criteria of dynamic stability, green water, and bow slamming (Table 5).

Table 4. The upper-level pairwise comparison using AHP method.

Upper-level objective	Fuel	Time	Safety
Fuel	1	3	1/6
Time	1/3	1	1/9
Safety	6	9	1

Table 5. The safety-related pairwise comparison using AHP method.

Safety criteria	Dynamic stability	Bow slamming	Green water
Dynamic stability	1	9	1/2
Bow slamming	1/9	1	1/8
Green water	2	8	1

In the next step, we provide a pairwise comparison between the intervals of each safety criteria separately. The relative importance of each interval is established and recorded in Table 6, where the assignment of priorities to each interval is based on expert knowledge and their analysis. It is important to highlight that as the safety parameter increases, the nonlinearity also increases, leading to priorities being determined in ascending order.

Table 6. The inner pairwise comparison for each of the safety issues using AHP method.

Safety criteria	Interval	1st	2nd	3rd	4th
Green water	1st	1	1/2	1/7	Not applicable
	2nd	2	1	1/5	Not applicable
	3rd	7	5	1	Not applicable
Bow slamming	1st	1	1/2	1/4	Not applicable
	2nd	2	1	1/3	Not applicable
	3rd	4	3	1	Not applicable
Dynamic stability	1st	1	1/5	1/6	1/9
	2nd	5	1	1/2	1/4
	3rd	6	2	1	1/5
	4th	9	4	5	1

It should be noted that the discretization approach used for the safety criteria is also employed for the inverse model. We conducted a discretization process on the safety-related data, using the intervals specified in Figure 7. Subsequently, each data point was multiplied by its respective interval weight as determined according to Table 6. Finally, we used the normalized values according to Equation (5) to ensure a standardized range of values between zero and 1.

In conjunction with the practical expertise of TNM, we also identified the best practice route that aligns with their current business strategy (Figure 8).

Figure 8. The best practice routes provided by TNM experts.

5 Results

The following section presents the results of implementing the proposed approach on the case study described in the previous section. We also include a sensitivity analysis on the impact of the number of best practice routes on the accuracy of weight results through a specific scenario of weights.

5.1 Inverse optimization

This section aims to assess the viability of regenerating the best practice routes using the proposed methodology. While such artificial weight settings are not typically available in practice, their inclusion in our case study is an important evaluation criterion. The model was tested across six weight scenarios as discussed earlier. Figure 9 illustrates the visual comparison between the predefined best practice routes and the regenerated ones. Moreover, the obtained weights and the associated objective values are indicated in Table 7 and Table 8, respectively. In the first scenario, the weight of voyage duration was identified as 0.948, and the green water weight was found to be 0.051. While there was a deviation in this case, the visual comparison revealed an almost negligible difference between the two routings. The deviation of the obtained weights from the initial ones arises from the existence of multiple possible solutions. In the subsequent run, the best practice routes were determined based on fuel consumption. The resulting weight was 0.978 for fuel consumption, and 0.022 for bow slamming and dynamic stability combined. Although there was a slight deviation from the target value, the well-aligned routings confirmed the marginal difference. Furthermore, there are multiple weight configurations that can yield the best practice routes.

The optimized weights concerning bow slamming resulted in the weights of 0.962 and 0.038 for bow slamming and green water, respectively, with no weights assigned to the other criteria. The green water criteria was more influential than bow slamming due to the synergistic relationship between their values, as demonstrated in Figure 5. In the subsequent scenario, where the best practice routes were formulated based on green water, the inverse optimization approach could identify the precise weight. A visual comparison of the two routings was performed by contrasting the best practice routes with the routing derived from the weight results, as depicted in Figure 9. The difference between the two routes was quite negligible. In general, there are multiple solutions that yield optimal routes, and the identified weights are associated with a single set of solutions with transportation costs nearly identical to those of the original best practice routes.

The subsequent weight setting examined the efficiency of the approach concerning the best practice routes defined based on the vessel's dynamic stability. The weight obtained for dynamic stability was 0.998, which was in close proximity to the target value of 1. Lastly, the weight of all objectives was set to be equal, and the results were satisfactory. Although the objective weights differed slightly from the initial weight setting, the optimized routes still aligned well with the best practice routes.

Overall, the findings of the applied method on the test cases validated the high degree of similarity between the best practice routes and the regenerated routes across a variety of weight configurations.

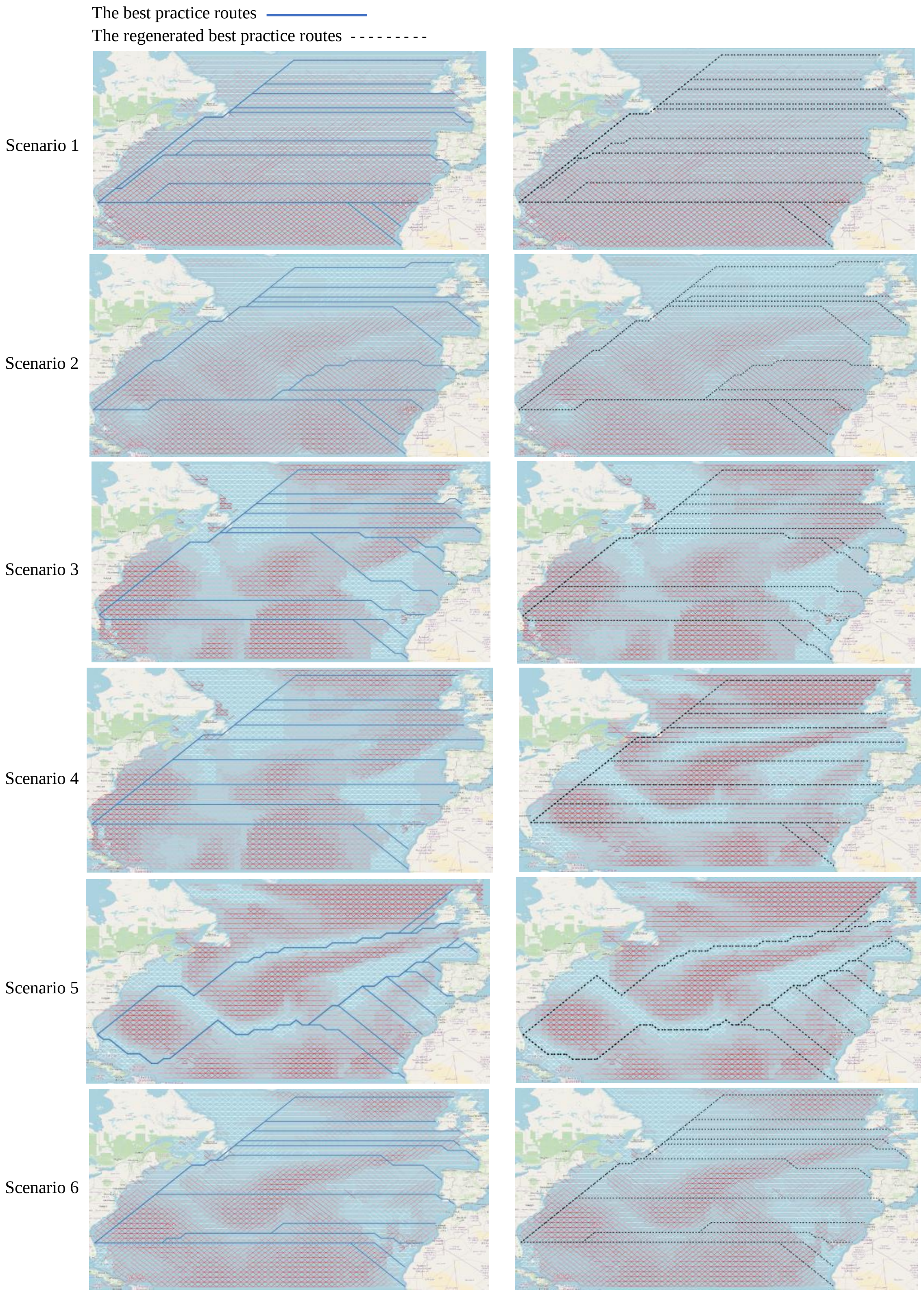


Figure 9. The visual comparison of the best practice routes with the routing derived from weight results.

Table 7. The weight result of the applied methodology on the case study.

Scenario	Optimization stage	Objective weight				
		Time	Fuel	Bow slamming	Green water	Dynamic stability
1	Before	1	0	0	0	0
	After	0.948	0	0	0.051	0.001
2	Before	0	1	0	0	0
	After	0	0.978	0.005	0.017	0
3	Before	0	0	1	0	0
	After	0	0	0.962	0.038	0
4	Before	0	0	0	1	0
	After	0	0	0	1	0
5	Before	0	0	0	0	1
	After	0	0	0	0.002	0.998
6	Before	0.2	0.2	0.2	0.2	0.2
	After	0.094	0.222	0.170	0.329	0.186

Table 8. The objective values and monetary cost of the route for each scenario.

Scenario	Route objective value			Route monetary value			Evaluation of criteria after optimization				
	Before	After	Change	Before	After	Change	Time	Fuel	Bow slamming	Green water	Dynamic stability
1	2,446,860	2,687,614	9.8%	$3,817,330	$3,817,770	0.0%	2,446,870	2,380,410	6,492,710	7,222,010	816,232
2	2,289,920	2,396,002	4.6%	$3,734,850	$3,735,640	0.0%	2,520,370	2,289,920	6,546,180	7,332,460	834,005
3	6,313,910	6,352,600	0.6%	$3,874,220	$3,866,950	-0.2%	2,535,250	2,466,560	6,315,810	7,286,230	768,108
4	7,208,200	7,208,200	0.0%	$3,850,660	$3,850,660	0.0%	2,458,700	2,430,390	6,464,150	7,208,200	806,442
5	394,010	411,080	4.3%	$4,586,970	$4,586,970	0.0%	3,525,660	3,421,540	7,887,810	8,677,770	394,010
6	3,847,550	4,369,270	13.6%	$3,786,090	$3,784,970	0.0%	2,480,800	2,358,200	6,370,730	7,251,550	779,350

Table 8 presents the detailed objective function values for the scenarios under consideration. It is important to note that to express the results in a more comprehensible format, we multiplied all values by 100. In Scenario 1, focused on minimizing travel time, we observed a 4% increase in fuel consumption. Conversely, in Scenario 2, which prioritizes fuel consumption efficiency, there was a 3% rise in voyage time. These findings imply that reducing route time leads to increased fuel consumption, while prioritizing fuel efficiency results in longer voyage times. Additionally, the scenario implemented to minimize bow slamming risk, by altering the route to avoid adverse weather conditions, led to a 7.7% rise in fuel consumption. The results highlight the inherent trade-offs in optimizing routing and underscore the importance of carefully evaluating the relative importance of each objective. DMs must weigh the costs and benefits of different strategies to achieve their goals while balancing multiple objectives.

As can be seen in the provided tables, although there was a difference between the weight settings before and after optimization, the objective function and the monetary cost of the routes approximately remained the same regarding the well-matched routings. More important, is that we reconstructed the best practice routes with the new weight settings. It is critical to note that we did not identify identical routes due to the existence of multiple solutions with similar cost. The objective function value differs due to the presence of multiple weight combinations that satisfy the optimality conditions. The objective function value differs as there are some margins of the weights for which the optimality conditions are satisfied.

The results presented in this study show that the proposed approach can effectively regenerate the best practice routes. Comparing the weight settings before and after optimization, we observed some differences, yet the total transportation cost decreased or remained approximately the same for the well-matched routings. Notably, we reconstructed the best practice routes with the obtained weight settings, which allowed us to evaluate the trade-offs between the different objectives. The results indicate that there were some differences in the optimized routes compared to the original routes. This discrepancy can be attributed to the existence of multiple sets of weight configurations that yield the same transportation cost.

5.2 A sensitivity analysis on the number of best practice routes

The present section aims to explore the impact of the number of best practice routes used as input on the accuracy of weight results. Specifically, we have selected scenario 6 to examine the weight outcomes of inverse optimization for 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20 best practice routes. In scenario 6, all objectives have equal weights, and the purpose is to evaluate how the determined weights change with an increase in the number of best practice routes. The weight outcomes obtained through inverse optimization are detailed in Table 9 and illustrated in Figure 10. The results clearly indicate that increasing the number of best practice routes leads to a more accurate determination of weight values. For instance, in a scenario with eight best practice routes, we observed an improvement in the weight results. The identified weights lead to enhanced fuel efficiency and dynamic stability, whereas the previous scenario yielded zero in these objectives. Furthermore, when comparing this scenario to another one with a larger number of best practice routes, specifically ten, the latter demonstrates a better capability of producing objective weights that are closer to the target value of 0.2. In general, in the context of our study, this resulted in weight values that gradually approached the target value of 0.2. Furthermore, the cost change drops dramatically from 88.6% at 2 best practice routes to 14.2% at 12 routes. However, beyond 12 best practice routes, this decrease becomes less pronounced, stabilizing around 14%, suggesting that increasing the number of routes beyond a certain point yields little additional benefit in terms of cost.

Table 9. The effect of the number of best practice routes on the objective weight estimation.

The number of best practice routes	Objective weight					Cost		Change (%)
	Time	Fuel	Bow slamming	Green water	Dynamic stability	Before	After	
2	0	0	0.0603	0.9397	0	676,786	1,276,550	88.6
4	0	0	0.0603	0.9397	0	1,321,160	2,510,000	90.0
6	0	0	0.5464	0.4536	0	2,001,060	3,539,920	76.9
8	0	0.3243	0.0829	0.5483	0.0445	2,720,230	3,769,940	38.6
10	0	0.2399	0.168	0.4174	0.1746	3,409,710	4,276,030	25.4
12	0.1019	0.2292	0.1781	0.3201	0.1707	4,133,570	4,719,870	14.2
14	0.0910	0.2225	0.1683	0.3320	0.1862	4,858,360	5,536,760	14.0
16	0.0937	0.2215	0.1702	0.3289	0.1856	5,569,230	6,334,090	13.7
18	0.0937	0.2215	0.1702	0.3289	0.1856	6,260,050	7,114,980	13.7
20	0.0937	0.2215	0.1702	0.3289	0.1856	6,973,130	7,916,170	13.5

The expansion of the destinations can be perceived as an effective strategy to diversify the best practice routes available. Inverse optimization, with increased information about feasible solutions, allows for the application of stricter constraints on potential solutions, resulting in higher-quality outcomes. As the level of detail on best practice routes increases, the problem becomes more constrained, leading to even better results. However, it is crucial to balance the amount of information with the complexity of the problem and the computational resources required to solve it.

Figure 10. The effect of the number of best practice routes on re-generating the routes.

5.3 A comparison between the AHP method and the inverse optimization approach

The effectiveness of the AHP and the inverse optimization methods in determining objective weights for multi-criteria problems is compared. Table 10 displays the objective weights derived from the AHP method, and Table 11 compares these weights with those determined through inverse optimization. Furthermore, Figure 11 visually presents the optimized routes achieved by applying the objective weights derived from both methods. When compared to the best practice routes, which serve as a feasible solution to model *P*, the inverse optimization model gave the highest weight to the safety issue related to green water. The AHP method also assigned a higher weight to this objective. The main difference between the two methods' weights lies in fuel consumption, where the route optimized by AHP was inconsistent with the best practice routes identified by experts.

According to Table 12, the evaluation of the two methods reveals notable findings:

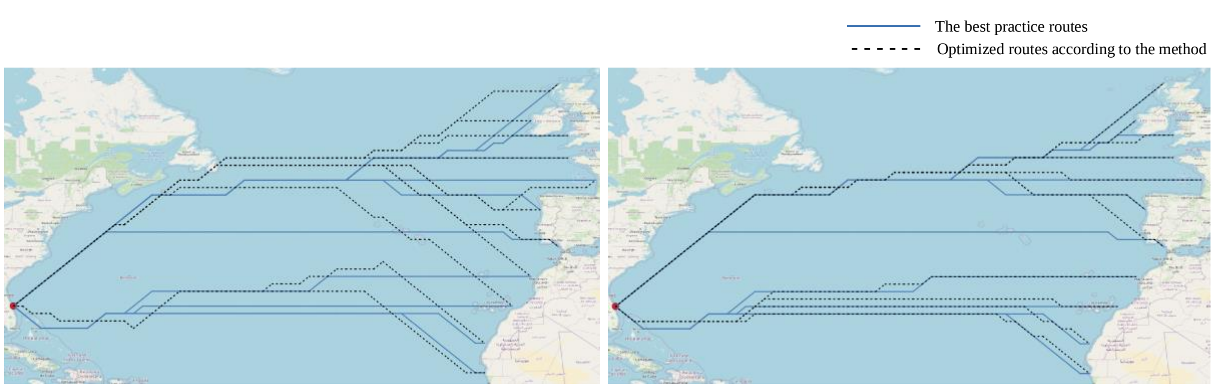
- The aggregate distance traveled in the context of inverse optimization routing exhibited a near identical measure to that of the best practice routes, whereas in the AHP method, it saw a marginal increase of 0.63%.
- In terms of fuel consumption, the routing based on the inverse model demonstrated closer alignment with the best practice routes, with a deviation of -2.21%, as opposed to the AHP method, which deviated by 4.25%.
- Similarly, with respect to travel time, the routing using the inverse optimization method exhibited closer proximity to the best practice routes, deviating by -0.80%, whereas the AHP method deviated by 2.55%.

Table 10. The final objective weights for multi-criteria route planning.

Objective		Weight	Total
Fuel		0.162	0.162
Time		0.068	0.068
Green water	1st interval	0.041	0.440
	2nd interval	0.073	
	3rd interval	0.325	
Bow slamming	1st interval	0.006	0.042
	2nd interval	0.010	
	3rd interval	0.026	
Dynamic stability	1st interval	0.012	0.288
	2nd interval	0.041	
	3rd interval	0.059	
	4th interval	0.176	

Table 11. The comparison between the weight results.

Objective	Obj. weight through AHP	Obj. weight through inverse optimization
Fuel	0.162	0
Time	0.068	0.020
Dynamic stability	0.288	0.369
Bow slamming	0.042	0.007
Green water	0.440	0.603



AHP

Inverse optimization

Figure 11. Comparison of the best practice routes (blue line) with the optimized route (dotted lines) through AHP and inverse optimization methods.

Table 12. The evaluation of the methods through criteria analysis.

Routing method	Total Distance	Evaluation of criteria				
		Fuel	Time	Bow slamming	Green water	Dynamic stability
Best Practice routes	77,981	2,675,080	2,693,840	6,881,110	7,604,000	572,616
AHP	78,475	2,561,320	2,762,590	6,671,220	7,613,670	670,055
Deviation (%)	0.63%	-4.25%	2.55%	-3.05%	0.13%	17.02%
Inverse optimization	76,974	2,615,840	2,672,270	6,756,200	7,505,360	497,303
Deviation (%)	-0.34%	-2.21%	-0.80%	-1.82%	-1.30%	-13.15%

6 Discussion

The developed methodology is a valuable contribution to the field of vessel route planning, as it provides high performance and flexibility compared to traditional methods. Specifically, our research suggests that inverse optimization leads to better routing decisions, closer to the best practice routes, than the AHP method.

Our methodology has the advantage of flexibility. If the objectives change or additional objectives are added, the process can be easily customized and re-applied more quickly than traditional resource-intensive methods. However, in scenarios with multiple origin and multiple destination points, the approach can be applied through the integration of multiple commodities. Such integration allows for defining distinct tree structures for each commodity.

One notable observation is that the number of destinations directly influences the weight results, as changes in the number of destinations leads to corresponding adjustments in the weights. By including more routes in the optimization process, we can achieve a more precise determination of the weights, thereby enhancing the accuracy of the overall routing solution.

In real-world applications, it is vital to involve domain expertise when defining best practice routes and determining the optimal weights that accurately represent them through inverse optimization. While this case study employed artificial best practice routes with objective weights in the minimum cost flow problem, incorporating expert knowledge ensures a more reliable representation of real-world scenarios. This process may be time-consuming, yet it is essential for ensuring accuracy and relevance. It is important to acknowledge that these best practice routes may exhibit occasional inconsistencies, which should be considered during the analysis. Often, the determination of best practice routes involves human judgment. Inaccuracies, oversights, or biases in these judgments can introduce inconsistencies.

Moreover, in this study, we assumed that the weather data remained static over the planning period due to the constant vessel speed. Incorporating dynamic weather patterns into a network can greatly increase the size of the model. Dynamic weather conditions can dramatically increase the computational resources and time required to solve the problem. This choice was motivated by the need to avoid complexities introduced by dynamic weather patterns, as the primary focus of the case study was to test our methodology. However, incorporating dynamic weather data that accounts for varying vessel speeds and corresponding time intervals can significantly improve safety management.

The safety concerns considered in this study exhibit non-linear characteristics. Although our methodology is proficient at determining objective weights even in the presence of non-linear behaviors, the ability to regulate these non-linear parameters provides an additional layer of control. Discretization methods offer a systematic approach to this regulation. By discretizing the function's values, we strategically partition the range of values into distinct intervals. This process involves categorizing data points based on their magnitudes and assigning them to the corresponding interval. Furthermore, to optimize each objective effectively, it is essential that the interval weights for each objective are assigned in a manner where higher intervals correspond to higher objective weights.

7 Conclusion

This paper introduces a novel approach to vessel route planning using inverse optimization, which includes the integration of multiple criteria into the weighted objective function. The proposed method enables DMs to identify their objectives and preferences based on observed outcomes related to safety, fuel consumption, and voyage duration, without the need for explicit criteria weighting. This approach offers an efficient and effective decision-making process that better aligns with DMs' priorities, making it a valuable contribution to vessel route planning. Our contributions encompass developing an inverse optimization model and conducting a comprehensive case study analysis using real weather data. These findings advance vessel route planning by incorporating several safety considerations and providing a methodology for determining optimal weights.

The proposed method demonstrates the efficiency of the model in reconstructing the best practice routes. By using inverse optimization, we effectively address the task of extracting implicit preferences from observed solutions. This empowers DMs to gain invaluable insights into the relative importance of objectives. Unlike previous techniques that relied on explicit weight values, our approach eliminates the need for predefined weights, which can lead to inaccurate weighting. This is especially significant as incorrect weights could result in misleading outcomes.

Future research can focus on prioritizing and integrating safety concerns into real-world vessel route planning scenarios. Such studies could explore the safety issues considered, the quantification methods employed by planners, and the trade-offs made between safety and economic factors. Additionally, optimizing route planning can be investigated, and the performance of the inverse optimization method be compared with other approaches using best practice routes provided by experienced professionals. In our case study, we relied on artificial best practice routes. In future research, it is advisable to adopt routes recommended by experts. This research can provide valuable insights for optimizing vessel route planning, achieving a balance between safety and efficiency, and benefiting maritime stakeholders by developing more effective and sustainable practices.

Acknowledgments

We would like to acknowledge the financial support provided by Université Laval, MITACS and PROMPT research organizations for this project.

Declarations of interest

The research work presented in this paper was conducted in a collaborative research partnership with the True North Marine company, which is based in Canada. This industrial partner provided the essential data for the case study presented in the paper.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT only to improve the English grammar and sentence structure.

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