Multi-objective Vessel Routing Problems with Safety Considerations: A Review

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Abstract

This paper provides a review of vessel route planning, with a focus on safety considerations and the complexity of multi-objective decision-making processes. This complexity arises from the difficulty of finding an appropriate balance between several objectives, often conflicting, that adequately reflects the preferences of the decision makers. The maritime industry faces the challenge of enhancing vessel route optimization for safety, operational efficiency, and cost-effectiveness. We describe quantitative methods to find routes that effectively balance multiple objectives, including safety, fuel consumption, and route duration. A significant focus is on the complexity of multi-criteria decision-making in this area, highlighting various methodologies for balancing the different objectives. Safety is critical in this context, involving a thorough consideration of navigational risks, environmental factors, and compliance with International Maritime Organization regulations. Specifically, we introduce quantitative approaches for

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integrating key safety aspects into the decision-making process, including dynamic stability, the probability of bow slamming, and the occurrence of green water.

Keywords: Maritime route optimization, maritime navigation safety, multi-objective decision-making, weighted sum method, weight estimation methods

1. Introduction

The maritime industry plays a vital role in international trade, facilitating the transportation of a significant portion of goods through sea routes (Yan et al., 2021). Therefore, the voyage route planning of vessels becomes essential. Before starting the navigation, one of the captain's concerns is selecting a preliminary route that aligns with multiple objectives. Fuel consumption and route duration undoubtedly hold significant importance, as optimizing these factors directly impacts operational efficiency and cost-effectiveness. Safety at sea is a primary concern and it encompasses protection from the risk of injury, navigational hazards, environmental threats and severe sea conditions like hurricanes, storms, high waves, and other extreme weather events. Maritime navigation safety depends on the vessel's characteristics, weather, and navigator expertise to minimize accidents and unforeseen events (Formela, 2019; Sánchez-Beaskoetxea et al., 2021), though it may increase fuel use and extend routes. In vessel route optimization, safety is ensured through hard and soft constraints (Kendall, 1975), which are defined in a mathematical routing model. Hard constraints safeguard the vessel by avoiding dangerous areas, while soft constraints allow routing flexibility, often with penalties for non-compliance. These constraints enable for route adjustments based on conditions and captain's judgment, seeking calmer seas to enhance vessel and cargo safety. Implementing these constraints relies on information about weather forecasts and vessel performance. For instance, to prevent capsizing in bad weather, a hard constraint keeps the vessel within a safe heel angle. A soft constraint, however, advises adjusting course or speed to mitigate rolling and increase stability. In optimization, hard constraints define the solution space, and soft constraints, when violated, lead to costs for maintaining feasibility, that can be managed by relaxing and penalizing the violation in the objective function. Hard constraints limit the solution space, whereas soft constraints can be integrated into the objective function.

The International Maritime Organization (IMO) regulations are instrumental in mitigating the environmental impact of maritime transportation, primarily through the enforcement of limits on sulphur emissions (Joung et al., 2020). Adhering to these rules is not only crucial for environmental sustainability but also for aligning with international maritime pollution control standards. In a significant regulatory shift in January 2020, the IMO mandated a reduction in the sulphur content in maritime fuels from a maximum of 3.5% to 0.5%, marking a substantial step towards cleaner maritime operations (Joung et al., 2020). In response to this mandate, the maritime community has introduced a variety of alternative fuels such as liquefied natural gas (LNG) and hydrogen to meet current and future emissions targets (Foretich et al., 2021). While this change is environmentally beneficial, it also poses new challenges in terms of fuel management and route optimization. The move to using low-sulphur fuel, potentially more costly due to its higher price, has resulted in increased expenses that have been transferred to ship-owners, yet they are not compensated by corresponding increases in the freight rates charged to customers. Therefore, companies have to cover these extra costs themselves, which adversely affects their profitability (Sigalas, 2022). Additionally, while the use of a particular fuel type may not pose significant operational challenges by itself, the complexity lies in maneuvering through various zones, each governed by different regulations. There is also an increased need for a careful balance between fuel consumption and time efficiency. Companies must optimize their routes not just for regulatory compliance, but also to ensure that fuel use is economically viable while meeting scheduling requirements. As a result, shipping companies, faced with increased fuel consumption costs, must develop innovative strategies to adapt to evolving standards while maintaining operational efficiency and profitability. Additionally, IMO has developed and adopted international regulations to prevent collisions and established comprehensive standards for seafarers, enhancing the safety of maritime voyages across the globe (IMO, 1972). Compliance with these safety guidelines is a pivotal concern of vessel route planning, ensuring that ships navigate safely through the oceans. The guidelines set by the IMO cover an extensive range of safety measures, crucial for the safe operation of ships in various maritime conditions. Given the complexity of these regulations and their effect on increasing operational costs, it is crucial for shipping companies to optimize navigation routes.

Optimizing vessel routes is influenced by factors like vessel characteristics and weather conditions. A ship's performance at sea depends on its size (length and width), load, engine, age, and design (Sánchez et al., 2021), which affects how it navigates through various environmental elements such as waves, winds, and currents. Routing algorithms play a pivotal role in identifying cost-efficient, energy-saving, and safe routes (Wang, 2018) by considering factors like distance, time, fuel consumption, and safety criteria, using data on weather conditions and vessel specifications. By leveraging these algorithms, planners can optimize routes, minimize transportation costs, reduce environmental impact, and enhance safety for crews, cargo, and vessels. Moreover, addressing safety involves multi-criteria problems to find an optimal route that balances safety, time, and fuel efficiency using a weighted sum framework. This approach requires flexibility to adapt to the dynamic aspects of navigation, aiming to ensure safety and cost-efficiency in route selection (Zhao et al., 2021).

In exploring quantitative route planning models and methods within the maritime industry, this paper seeks to uncover the sophisticated approaches used to integrate safety considerations. Specifically, we will delve into how safety is quantified and incorporated into existing models, and the methodologies that prioritize safety within multi-objective planning. To structure our research and provide a focused narrative, we have established these main research questions that will be addressed throughout this paper:

I. What existing methodologies in multi-objective planning address the identification of trade-offs among conflicting objectives, and how do they incorporate objective priorities and weights to achieve optimal planning solutions?

II. How prior studies have quantified and integrated safety issues into maritime routing problems, and what methodologies have been employed to formulate these safety considerations to enhance decision-making in maritime route planning?

III. How the field of vessel route optimization tackle the challenges related to safety considerations, and what methodologies are used to determine objective weights in scenarios involving weighted multi-objective problems?

A database search strategy was devised using the keywords listed below, combined with the "OR" operator, to address each of the research questions defined above. It was employed for searches across multiple platforms, including Google Scholar, ScienceDirect, IEEE Xplore, Scopus, and Web of Science.

- Keywords for research question I: Multi-objective optimization techniques, weighted sum method, multi-criteria decision making, objective weight estimation methods, exact methods, heuristic algorithms.
- Keywords for research question II: Vessel navigation safety, ship routing safety, maritime safety regulations, voyage safety guidelines, vessel routing safety.
- Keywords for research question III: Ship voyage optimization, safe vessel routing, vessel weather routing, ship safe route optimization, severe weather vessel routing, multi-objective weather routing, multi-criteria vessel routing.

In this review paper, we make three contributions to the field of maritime routing with safety considerations. First, we explore methodologies for establishing objective weights in weighted multi-objective problems, offering insights critical for decision makers (DMs) in the maritime industry. Second, we conduct a review of existing studies addressing safety concerns in maritime routing and provide a specific quantification of certain safety issues. Third, we discuss advancements in vessel route optimization with a focus on safety, highlighting novel methods and technologies for weather-related challenges in maritime route planning.

The subsequent sections of this paper are structured to delve deeper into each of the research questions. In Section 2 we explore methods for multi-criteria decision-making, with a particular focus on weight estimation approaches within the weighted sum method. It sheds light on the techniques utilized to make informed decisions in complex routing scenarios. In Section 3, we investigate safety considerations within maritime routing problems, and offer specific methodologies for quantifying certain safety concerns. Section 4 delves into innovations in vessel route optimization, with a primary focus on safety. This section highlights novel methods and advanced technologies to address the challenges posed by weather-related factors in maritime route planning. Finally, in Section 5, our concluding remarks provide a summary of the key findings from these sections, with a focus on highlighting the importance of safety and the application of multi-objective programming in the ever-changing domain of maritime routing.

2. Multi-objective planning

In multi-criteria optimization problems, finding the appropriate trade-off between the objectives poses a significant challenge. With multiple conflicting criteria to consider, DMs must carefully balance conflicting objectives to arrive at an optimal solution. The complexity arises from the fact that each objective, measured in different units, may have different priorities or weights, and changes in one objective can affect others. Additionally, quantifying or assessing the inherent trade-offs and independencies between objectives can be challenging. Introducing safety concerns adds another layer of complexity into the optimization task. Unlike objectives that can be directly associated with monetary values, such as fuel consumption and route time, safety issues are often evaluated through risk assessment models. This creates a significant discrepancy, underscoring the challenge of achieving a balanced integration of these objectives. It highlights the necessity for sophisticated optimization techniques that can adeptly navigate the complicated trade-offs between cost efficiency and enhanced safety, ensuring that neither aspect is compromised.

The route planning problem involves identifying the most efficient path. This entails constructing a network of nodes and arcs. The nodes serve a dual purpose: physically, they represent geographical points of interest such as ports and navigational landmarks, while temporally, they include dynamic information such as travel times, which vary with distance, ship speed, and weather conditions. This ensures that routing decisions account for both spatial and temporal factors. Arcs, on the other hand, define the feasible paths or connections between these nodes, forming the backbone of the routing network. They represent critical navigational parameters such as weather conditions, distance, the direction of travel, ship speed, and engine RPM (a measure of the engine's rotations per minute) necessary for various speeds.

2.1. Weighted sum method

The weighted sum method simplifies the complexity of the multi-objective optimization by converting several objectives into a single one, through the use of weights that reflect the priorities of DMs. Introduced by Zadeh in 1963, it has been widely discussed and applied across a variety of fields, as highlighted by Cohon (1983) and Odu and Charles-Owaba (2013). This method allows users to tailor their decision-making process according to their individual preferences. However, this advantage demands a deep understanding of the implications of each objective and navigating the trade-offs among them. Despite its straightforward approach, the method introduces challenges such as potential bias and the need for DMs to consider a wide range of factors, including ethical and regulatory limits.

To enhance the effectiveness of the weighted sum method, it is imperative to assign accurate weights to various objectives. Hwang and Masud (2012) highlighted the potential for bias in results due to incorrect weighting, underscoring the difficulty in establishing weights that truly reflect the intentions of DMs. This step is crucial to ensure that the outcomes are in line with their priorities. Recognizing this challenge, Odu (2019) documented the development of diverse techniques for determining these weights. These techniques are categorized into subjective and objective methods. Subjective methods rely on the individual's preferences, judgments, and values, while objective methods leverage quantifiable data for decision making. This categorization provides DMs with a range of strategies to effectively balance and prioritize their objectives. In the remainder of this section, we conduct an analysis that contrasts objective and subjective approaches, evaluating their efficacy in addressing the decision-making challenges present in multi-objective vessel routing problems.

In multi-criteria decision-making, several subjective approaches such as the point allocation, direct rating, ranking, swing weighting, nominal group technique, and the Delphi method offer diverse strategies for evaluating decision criteria. Doyle et al. (1997) introduced the point allocation approach, providing a simple and effective technique for assessing the importance of various criteria in decision-making processes. Its simplicity lies in the straightforward distribution of points, allowing the DM to easily perceive the relative significance of each criterion. When a criterion is assigned more points, it symbolically highlights its elevated importance. By standardizing to a total of 100 points, the methodology ensures that the criteria weights are easily comparable and normalized. However, it's essential to note its limitations. The precision level offered by this method might not be ideal for more complicated decision-making processes. Moreover, while the approach ensures normalized and comparable criteria weights, it might not adequately capture non-linear or interdependent relationships between criteria, which can be crucial in certain contexts. Additionally, when the DM deals with five or more criteria, the method's simplicity becomes compromised, leading to a more complex allocation process. Given its complexity and lack of precision, the point allocation method may not be the ideal approach for handling multiple objectives.

Arbel (1989) proposed the direct rating as another subjective method, offering a distinct approach to criteria weighting. In this method, the DM is tasked with ranking the criteria based on their respective importance. Unlike fixed point scoring techniques, the direct rating method provides greater flexibility. It allows the DM to modify the importance of a particular criterion without necessitating adjustments to the weights of others. This inherent flexibility can be particularly advantageous in scenarios where the relationships between criteria are not strictly proportional or linear. However, the limitations of the direct rating method are similar to those of the point allocation method.

The ranking technique outlined by Roszkowska (2013) is another subjective method. In this method, typically, criteria are organized in a hierarchy, and ranked from the most important to the least. Subsequently, the ranks are transformed into weights through the utilization of one of three distinct methods: rank sum, rank exponent, or rank reciprocal. For the rank sum approach, weights are deduced from individual ranks and then normalized by dividing them by the total sum of all ranks. The rank exponent method, while similar in concept to the rank sum differs in that the value is raised to an exponential corresponding to its assigned rank. Lastly, the rank reciprocal method relies on utilizing the inverse of the criterion rank, ensuring the weight is the normalized reciprocal of that rank. Each method offers its unique perspective on emphasizing the importance of criteria, suited for different decision-making contexts. A disadvantage of this approach is its reliance on the subjective judgment of DMs to establish criteria hierarchy and assign ranks can introduce bias and inconsistency into the process. This subjectivity may result in imprecise weight assignment, potentially affecting the overall quality of the decision-making outcomes. Furthermore, the three rank transformer approaches may yield varying results in different decision scenarios, making it challenging to ensure consistent application. Additionally, this method might not always catch the small but crucial differences in criteria, especially in complex decision-making scenarios.

Parnell and Trainor (2009) introduced the swing weighting method as a structured technique for evaluating decisions. This approach begins with the selection of initial objective weights and then focusing on those objectives with least significant. Decision-makers then determine an objective they consider capable of swinging from the least to the most significant. This becomes the reference objective, usually assigned the ideal weight of 1 to symbolize optimal desirability. Subsequent steps involve comparing the importance of remaining criteria to this reference, assigning them values on a scale from 0 to 1. To maintain balance and fairness, a normalization step adjusts these weights so their total equals 1. A disadvantage of this method is its dependency on the subjective determination of the reference objective, a process potentially vulnerable to bias. The method's success depends on the decision-maker's precision in evaluating other criteria against the reference, a nuanced and error-prone task.

The nominal group technique, as described in Abdullah and Islam (2011), serves as a structured brainstorming approach. This method is designed to generate a wealth of ideas around a given topic, and ensure that every group member has an equal voice in the process. Beyond its capacity for idea generation, the nominal group excels in the prioritization of these concepts. One of its standout features is the democratic nature of idea selection: ideas with the most votes are prioritized. This approach ensures a balanced reflection of group opinions. However, it can be time-consuming, especially for complex problems, and resource-intensive. The individual tendency to comply with the majority opinion may arise, innovation could be limited, and it is most suitable for smaller groups due to management challenges with larger ones. The scoring process may become complex with multiple criteria, possibly diverting attention from in-depth discussion. Strong personalities within the group can also influence outcomes, potentially overshadowing quieter participants. When employing nominal group for decision-making, these drawbacks should be carefully balanced against its advantages.

The Delphi method, developed by Dalkey and Helmer (1963), utilizes the collective expertise of a panel to determine the relative importance of various criteria in multi-objective decision-making (Rowe and Wright, 1999). Each expert independently assesses the significance of various criteria, and these evaluations are subsequently refined through multiple rounds of review and discussion in the process. This iterative approach ensures that individual assessments evolve in response to the collective insights of the group. The Delphi method typically leads to more reliable results than individual opinions or unstructured group decisions, as its core focus is on harmonizing diverse experts' opinions. Additionally, the structured design of this approach, along with ensuring participant anonymity, minimizes bias and the impact of more assertive voices. Although the method is well-structured, its reliance on subjective inputs means that the outcomes might not fully represent objective viewpoints or the wider perspectives beyond the expertise of the panel. Furthermore, the lack of face-to-face interaction may restrict the extent of discussion and comprehension that is often possible in more interactive environments.

In overall, subjective approaches in decision-making offer unique advantages, especially in their ability to incorporate qualitative aspects, expert judgments, and implicit insights that cannot be easily quantified. This flexibility is particularly crucial for effectively addressing complex or multifaceted issues. Furthermore, these methods facilitate the decision-making process by eliminating the need for exhaustive data analysis and complex computations, while also embracing a diversity of perspectives and values. This approach promotes stakeholder engagement and ensures that decisions are in line with organizational goals. However, these methods are not without their challenges. One significant concern is the potential for bias and inconsistency, as personal preferences and emotional influences might twist weight assignments unfairly. Such subjectivity undermines transparency, thus complicating the rationale behind decisions. Additionally, the issue of reproducibility arises, as different decision-makers may assign diverse weights to identical criteria, potentially compromising the reliability of the process. The absence of a normalization method means that applying the same decision-making framework could result in varying outcomes, questioning the method's credibility. Moreover, the task of accurately assigning weights through subjective evaluation introduces further complexity, potentially compromising the quality of decision-making outcomes. This complex interplay between the benefits and limitations of subjective methods underscores the importance of carefully balancing these factors to enhance decision quality.

The Analytic Hierarchy Process (AHP), entropy weighting, standard deviation, and statistical variance are objective decision-making approaches. The AHP (Saaty, 1980) simplifies complex decision-making by breaking down a decision into simpler, smaller components. It structures the decision problem hierarchically, sorting criteria logically. Through pairwise comparisons, it quantifies the relative significance of these criteria, allowing for weighted decision-making. While the comparisons rely on subjective judgment, the objective nature of AHP is highlighted through the use of consistency ratio checks. These checks assess the reliability and uniformity of the pairwise comparisons made by DMs, identifying any inconsistencies or contradictions. The consistency ratio provides a quantitative measure to determine the alignment of comparisons. By evaluating this ratio against a predefined threshold, AHP ascertains the decision process's consistency level. However, the reliance on subjective judgment in pairwise comparisons introduces potential biases, originating from individual perceptions and experiences. These biases could affect the precision and reliability of AHP outcomes. Moreover, as the complexity and number of objectives increase, the AHP process becomes more resource-intensive and challenging, necessitating expertise in its application. This complexity increases with each added objective, magnifying the need for a comprehensive re-evaluation of comparisons, which may deter its application in certain scenarios due to the increased complexity and resource demands.

The entropy weighting method is a sophisticated tool for assigning weights objectively in multi-criteria decision-making, leveraging data variability to prioritize criteria (Zou et al., 2006). Originating from information theory, this method assesses the variability or unpredictability within criteria across various options. It begins with linear normalization data to ensure uniformity across all values. A matrix is then established, representing alternatives as rows and criteria as columns, showcasing how each alternative, specifically routes, performs based on the criteria. Entropy is computed for each criterion to determine its randomness across the alternatives, where the diversification level for each criterion is calculated by subtracting its entropy value from one. Criteria demonstrating with higher diversification is associated with lower entropy values, suggesting greater importance. This reflects the principle that criteria exhibiting more variability are pivotal in decision-making. Parallel to this, the standard deviation/statistical variance approach, highlighted by Odu (2019), uses the same matrix but focuses on the dispersion measures of the data, providing an alternate perspective for determining criteria importance. However, these objective frameworks, while robust, have their limitations. They assume a direct correlation between data behavior and its significance, which might not always hold true in practical scenarios. Moreover, they might fail to capture the nuanced preferences of decision-makers. Additionally, these methods heavily rely on the quality of the data, any inaccuracies or incompleteness in data can change the outcomes. These considerations highlight the necessity of careful data handling and the integration of subjective insights to complement these objective approaches in decision-making processes.

Inverse optimization (Burton and Toint, 1992), offers a unique approach to decision-making by reconstructing the preferences of a decision-maker based on observed solutions rather than starting with explicit weights. Inverse optimization begins with the observed solutions and traces back to discover the underlying preferences that produced these solutions. It adheres to the KKT conditions of optimality established by Kuhn and Tucker (1951). This method is especially beneficial when the decision-maker’s goals or preferences are unclear or hard to specify directly. It eliminates the need for decision-makers to explicitly assign weights to each criterion, which may be unclear or hard to specify directly, thereby simplifying and speeding up the process. This flexibility allows for quick adjustments if objectives evolve or if new ones are introduced, making it a more adaptable solution than traditional approaches. However, the effectiveness of inverse optimization is significantly influenced by the availability of a diverse range of solutions to serve as a foundation and the quality of the observed solutions. If these solutions are incomplete or fail to accurately represent the preferences, the objectives derived from them may be misleading or incorrect.

2.2. Exact methods for path optimization

A variety of algorithms have been developed with the goal of identifying the most efficient and safe path, thereby achieving optimal solutions. Among these, algorithms like Dijkstra’s algorithm and A* can find the shortest path given a cost associated with each arc, that can be influenced by various factors such as travel time, distance, vessel speed, fuel consumption, and safety.

Dijkstra’s algorithm (1959) is a very common method for finding the shortest paths in a graph with non-negative weights. The A* algorithm (Hart et al., 1968) extends Dijkstra's algorithm by incorporating heuristic functions to direct the search towards the destination node. The result is the shortest path to a subset of the nodes, including the destination node, thereby enhancing solution time.

The isochrone method (James, 1957) is a strategic approach used in various fields, such as maritime navigation, urban planning, and emergency response, to optimize routes and resource allocation based on travel time. This method involves plotting isochrones, which are lines connecting points that can be reached within the same time frame from a specific starting location, given certain conditions or constraints. It begins by identifying all potential destinations that can be reached within the first time interval, considering variables such as speed and environmental conditions. These destinations form the first isochrone, essentially a boundary of equal travel time. With each iteration, the method extends from the previous isochrone, generating new boundaries that represent further distances achievable in the next time interval under the same conditions. This sequence of expanding isochrones visually represents how far one can travel over time, accounting for constraints like speed limits, geographical barriers, or varying conditions for movement.

2.3. Pareto optimality

A fundamental concept in multi-objective optimization problems is Pareto optimality. It defines a specific subset of solutions, where it is impossible to improve one objective without negatively affecting at least one other objective (Coello, 2010). Cohon (1983) developed the concept of Pareto optimality, which is a significant contribution to the field of multi-objective optimization. The generation of the Pareto frontier in a weighted multi-objective vessel routing problem follows a systematic procedure. Firstly, objectives are identified. Then, an objective function is formulated by scalarizing the objectives with weights, reflecting their relative importance. Subsequently, all possible weight settings are established, and the optimization problem is solved to identify optimal solutions for each specific weight configuration. Non-dominated solutions, which cannot be improved upon across all objectives at once, are gathered. These solutions ultimately define the Pareto frontier, illustrating the trade-offs between objectives. DMs can then make informed choices from the frontier based on their preferences.

Martins' labelling algorithm, introduced by Martins (1984), aims to identify Pareto optimal paths. This method is a technique for determining the most efficient path between two nodes in a graph with non-negative edge weights. Each node is assigned a set of labels, with each label representing a potential path from the starting node and including a vector of values that corresponds to the multi-dimensional costs of the path. The algorithm begins with an initial label at the source node and expands through the graph. Labels are added to nodes based on non-dominance, with new labels reflecting the total cost of reaching that node via a specific path. Dominated labels are removed, ensuring only Pareto optimal paths are maintained. The algorithm proceeds by updating labels across the graph until all Pareto optimal paths from the source to other nodes are identified.

2.4. Meta-heuristics

Multi-objective evolutionary algorithms aim to provide a set of well-approximated Pareto-optimal solutions. Such approach begins by generating an initial population of potential feasible routes known as individuals or candidate solutions. These individuals can be produced through various methods including random selection, a structured method like the shortest distance routes, or by drawing a straight line connecting the origin to the destination. The individuals are then combined using crossover and modified through mutation to create a new generation of solutions. Crossover is a crucial operator that combines individuals, referred to as parents, to generate one or more new individuals, aiming to explore the solution space. This process, by blending attributes from parent individuals, helps in maintaining diversity and spreading the beneficial characteristics, thereby supporting the gradual progress towards optimal or near-optimal solutions. On the other hand, mutation introduces small random changes to the information of individuals in the population, which may include adding or removing nodes, swapping nodes between routes, or adding/removing edges in the path. Mutation prevents premature convergence to suboptimal solutions within the algorithm and introduces diversity into the population, facilitating the exploration of unexplored areas in the solution space that may be missed by crossover alone. This iterative process continues until a defined stopping condition is met. Stopping conditions can include factors such as the maximum number of generations, a specific running time, or the fulfillment of predefined criteria that lead to a satisfactory Pareto front approximation. However, it is important to note that they might not always find the exact Pareto optimal front, especially in complex and multi-objective scenarios. The Multi-Criteria Evolutionary Weather Routing Algorithm (MEWRA) is an evolutionary algorithm designed to find the most efficient maritime route while considering conflicting factors like safety, time, and fuel consumption (Szłapczynska and Smierzchalski, 2009).

Genetic Algorithms (GA), developed by Holland (1992), are sophisticated evolutionary methods inspired by the principles of natural selection and biological evolution. Their core purpose is to address complex search and optimization problems. The procedure starts with a diverse set of potential solutions, typically encoded as strings. These candidates are assessed using a fitness function to determine their problem-solving effectiveness. The most effective solutions are chosen to breed, involving crossover for sharing genetic information and generating new offspring. To enhance genetic variety and premature convergence on suboptimal solutions, mutations are randomly applied to some individuals. As generations progress, the population gradually evolves to better solutions. The algorithm stops when it meets specific criteria, such as a maximum number of generations or an acceptable solution quality.

Particle Swarm Optimization (PSO), introduced by Eberhart and Kennedy (1995), is a heuristic algorithm that takes inspiration from the natural group behaviors observed in birds and fish. This iterative method starts with a group of random particles, each representing a potential route to an optimization problem. During the process, these particles move through the solution space, guided by their own best positions and those of their peers or the whole group. This dynamic ensures a combination of personal exploration and shared knowledge. PSO is unique in how it updates the movement and speed of the particles, influenced by each particle's individual best outcome and the best solution found by the entire group. Through a series of iterative steps, the algorithm modifies the particles' velocities and positions, systematically guiding them towards the optimal or nearly optimal solutions.

Ant Colony Optimization (ACO), developed by Colorni et al. (1991), is a heuristic algorithm inspired by the foraging behaviors of ants. It builds on how ants efficiently discover and follow the shortest routes using pheromone trails. In ACO, artificial ants simulate path exploration and release virtual pheromones to mark paths leading to favorable solutions, similar to real ants. Over time, the most traveled paths emerge as the most optimal solutions, making ACO effective for solving complex optimization problems.

3. Safety issues in maritime routing

In adverse weather conditions, a ship may encounter various hazardous phenomena, potentially jeopardizing the vessel's stability, damaging cargo and equipment, or posing risks to individuals on board. The extent of these dangers can vary significantly between vessels, depending on factors such as the type and size of the vessel, its age, average speed (Aalberg et al., 2022), hull design, and stability parameters. Therefore, in the context of vessel route planning, it is crucial to take into account diverse safety issues. The safety of a vessel can be evaluated from various perspectives, encompassing a broad range of aspects that adhere to regulations set forth by organizations like IMO (Fabbri and Vicen-Bueno, 2019). Additionally, seakeeping criteria play a significant role in assessing vessel safety and will be further examined in detail. Several crucial factors that affect safety, fuel consumption, and route time in maritime transportation can be assessed by considering wind conditions (direction and speed) and wave characteristics (height, direction, and frequency). For instance, if a vessel, operating under the same engine settings as in calm water conditions, encounters a headwind, it would experience a decrease in speed and an increase in fuel consumption. Similarly, the presence of large waves on a particular side can compromise the stability of the vessel.

Seakeeping is one of the safety concerns essential for ensuring a ship's operational effectiveness and safety under different sea conditions. This aspect is particularly critical when optimizing vessel routes with safety considerations. By assessing and predicting how a vessel will perform in response to environmental factors like waves, wind, and currents, seakeeping ensures the ship remains stable and safe. Such evaluations are key to route optimization, allowing for the selection of paths that avoid severe weather and challenging sea states. Seakeeping involves theoretical analysis that utilizes mathematical models and equations to understand the complex interactions between fluids and structures in the vessel industry (Pennino, 2020). As illustrated in Figure 1, the behavior of vessels can be classified into six motion behaviors: three linear motions along longitudinal, transverse, and vertical axes (surge, sway, and heave), and three rotational motions along these axes (roll, pitch, and yaw). These behaviors are referred to as seakeeping characteristics. To accurately predict a ship's behavior in response to external forces like waves, currents, and wind, naval engineers consider various essential factors. These factors include the shape of the vessel, its motion characteristics in water, and the distribution of its weight. By applying principles derived from mechanics and fluid dynamics, engineers can comprehensively anticipate and forecast how the ship will respond to these external forces. This process involves developing mathematical models and equations that capture the complex dynamics of the ship in a marine environment.

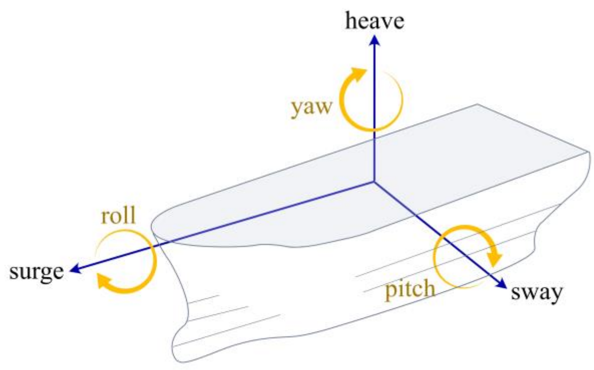

Figure 1. Six fundamental motion behaviors of a ship.

Ship speed is one of the most critical factors affecting vessel safety when it comes to managing safety concerns associated with ship motions. Calculating a ship's speed in various weather conditions involves evaluating factors such as wind speed and direction, wave height, sea state, and ocean currents. Optimizing ship speeds under varying weather conditions is crucial not only for safety but also for reducing fuel consumption, where changes in weather conditions significantly influence the optimal speed strategy for maritime routes (Ormevik et al., 2023). Ship speed reduces in hazardous weather due to the added resistance introduced by waves, wind, and ship motions. The interaction between the ship's hull and the surrounding water leads to additional resistance to the ship's movement, known as ship-added resistance. Various elements, including the height of the waves, the speed of the ship, its orientation towards the wave, the design of the hull, and the general conditions of the sea, contribute to this added resistance. The ship's propulsion system must overcome the added resistance to its forward movement caused by this interaction. This resistance is crucial in sea navigation, affecting fuel consumption and the ship's total efficiency.

3.1. Stability

Ship stability is crucial for maritime safety and efficiency. It encompasses two main areas: intact stability, which is the ship's ability to remain upright in calm conditions, and dynamic stability, concerning the ship's response to rough seas and strong winds. Both aspects ensure a vessel's safe navigation and resilience in various marine environments. Building on the fundamental understanding of ship stability, it is crucial to delve into the key concepts that form the basis of stability criteria. These concepts are graphically illustrated in Figure 2, showcasing the interplay of the center of gravity (G), metacenter (M), and buoyancy (B) that are pivotal elements in naval architecture. The center of gravity is the distributed weight of a vessel over its length for which its force (F_G) acts downward. The center of buoyancy is a geometric center of the underwater volume of a vessel with an upward force (F_B). The metacenter is the intersection point of the vertical line associated with the force of buoyancy and the center line (Lee, 2019).

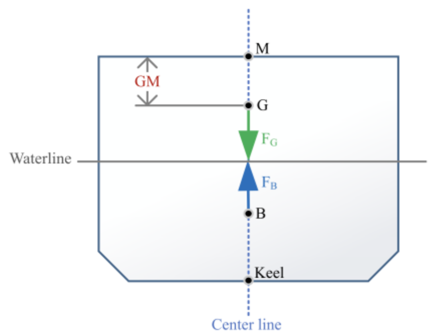

Figure 2. The position of metacenter, center of gravity and buoyancy for floating body of a vessel.

In general, the metacenter and the center of gravity are located on the center line of a vessel. Moreover, the position of the keel of a vessel is another important aspect regarding the definition of the vertical distance from the center of each of the abovementioned centers to another, for which the critical one is GM. It is the distance between the center of gravity and the metacenter of the vessel. When an external force tilts a vessel, the position of the buoyancy centre shifts as the vessel's underwater volume changes. The position of the metacenter against the center of gravity represents the vessel's stability; when the metacenter is above the gravity center (so-called positive metacentric height), the vessel can return to the upright position, tilting by external forces. In contrast, the situation in which the center of gravity is placed above the metacenter results in the vessel's capsizing. Moreover, the coinciding of the two centers leads the vessel to remain in the tilted position, the so-called equilibrium status (Figure 3). Hence, the larger GM represents greater stability against overturning.

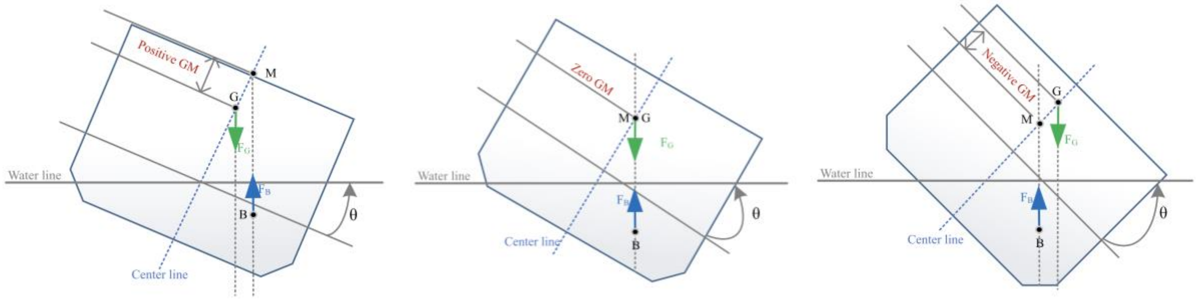

Figure 3. Stable equilibrium (left), neutral equilibrium (middle), and unstable equilibrium (right) of floating body of a vessel.

The transverse distance between the center of gravity and buoyancy is called GZ. By plotting the GZ value against the heeling angle (θ) using the law of sines (Equation 1), the GZ curve is obtained (Lee, 2019). An example of such a curve is depicted in Figure 4. The modelling of stability criteria depends on the area under the GZ curve using an integral function representing the ship's transverse statical stability.

$$GZ(\theta) = GM. \sin\theta \quad (1)$$

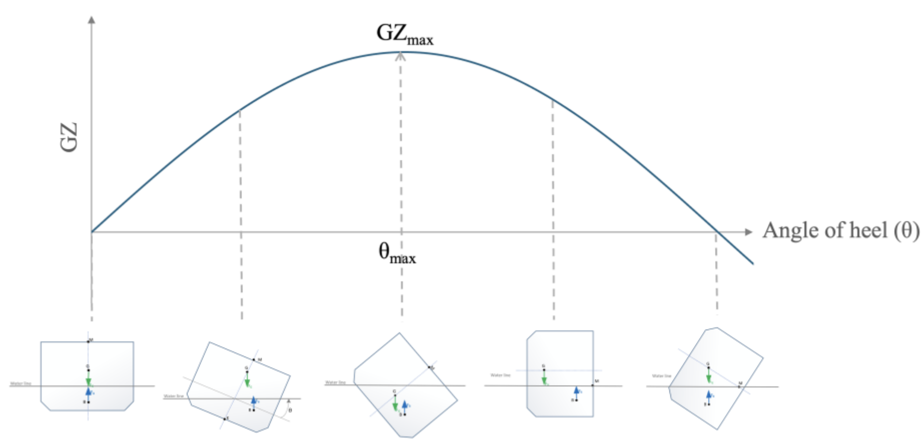

Figure 4. An example of GZ Curve and associated vessel stability through the position of buoyancy and gravity forces across different heel angles.

GM is calculated by determining the distance from the keel to the metacenter (KM) and subtracting the distance from the keel to the center of gravity (KG), as represented in Equation (2). The relative steps to calculate KM and KG are described in Appendix A.

$$GM = KM - KG \qquad (2)$$

3.1.1. Intact stability criterion

The intact stability criterion is the initial condition of a surface ship on calm water, which is affected by GZ curve characteristics. The ship's survivability on the ocean can be threatened by insufficient stability (Alberto, 2016). The stability criteria of a vessel on calm water are straightforward according to the area enclosed by the GZ curve up to the heeling angle of 30° (area A_1, in meter-radians or m.rad) and 40° (area A_2, in m.rad), as illustrated in Figure 5. Based on IMO guidelines (DNV GL, 2019), the intact stability condition of a vessel depends on its GZ curve specifications. The criteria are as follows:

1- The area under the GZ curve up to 30° (area A_1) should be at least 0.055 m.rad.

2- The area under the GZ curve between 30° and 40° (area A_2) should be at least 0.03 m.rad.

3- The total area of A_1 and A_2 must be at least 0.09 m.rad.

4- The total area under GZ above 30° must be larger than or equal to 0.2 m.rad.

5- The angle of the heel in which the maximum GZ is reached (θ_{max}) should be at least 30°.

6- The GM should be more than 0.15 m.

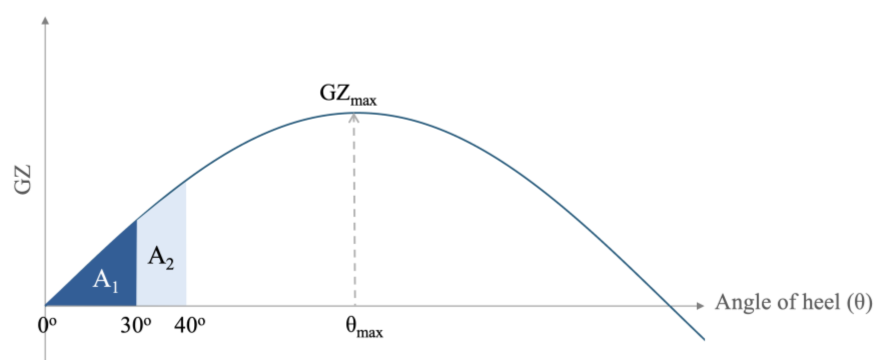

Figure 5. Illustration of areas *A1* and *A2* within GZ curve for intact stability evaluation.

The vessel's intact stability may be compromised if it fails to satisfy any of the above criteria.

3.1.2. Dynamic stability criterion

The dynamic stability criterion is formulated by focusing on wind as the primary external force that causes ships to tilt (DNV GL, 2019). Figure 6 demonstrates the magnitude force of the wind that causes the ship to tilt (Area *a*) and the ability of the ship to resist the imposed pressure (Area *b*). Therefore, the vessel would be in stable equilibrium as long as Area *b* is bigger than Area *a* (Equation 3).

$$s_D = \frac{Area\ a}{Area\ b} \leq 1 \qquad (3)$$

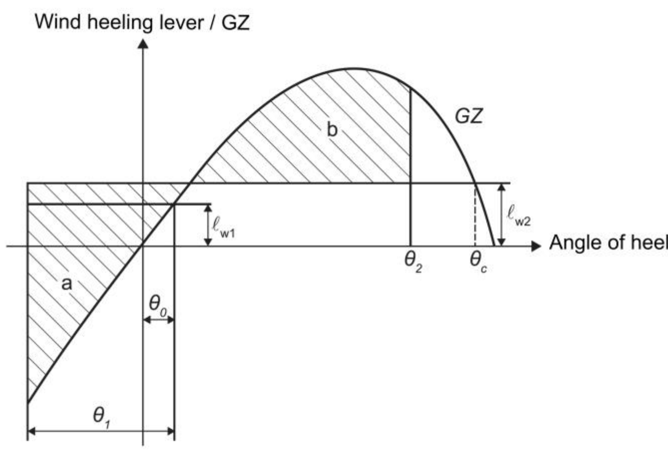

Figure 6. Graphical representation of wind force magnitude causing ship tilt (Area *a*) and ship's righting moment resistance (Area *b*) for dynamic stability evaluation. Adapted from DNV GL (2019).

The description of the various angles illustrated in Figure 6 includes θ_0, which is the angle of the heel under the action of steady wind; θ_1, representing the angle of the roll to windward due to wave action; and θ_2, defined as the smaller angle between down flooding, 50° or θ_c. The wind heeling lever indicates the capability of wind to induce a tilt in a ship, where l_{w1} and l_{w2} correspond to steady and gust wind, respectively. They are constant values at all angles of inclination and can be calculated as described in Appendix B.

To demonstrate the practical implementation of proposed dynamic stability criteria, a specific example will be explored. Consider a ship with a GM of 3.59 meters and a displacement mass of 87,337 tons, in a scenario where the vessel encounters a wind speed of 15 knots from a direction of 40 degrees and is moving at a speed of 12 knots in a direction of 0 degrees. The approximate Area a and b are 0.744 and 0.755 m.rad, respectively. These two areas yield a ratio of 0.986, which is less than 1, therefore indicating that navigation under these conditions would be safe for the vessel.

Now, we will examine another scenario where the wind speed is 41 knots with a direction of zero degrees, while the ship is moving at 12 knots in a direction of 90 degrees. The calculated Area a is 0.745 m.rad, and the Area b is 0.7449 m.rad. The resulting ratio in this case is 1.0001, which is above 1. Consequently, this indicates that the vessel is at risk of capsizing. Therefore, areas with such scenario should be removed from the solution space.

3.2. Bow slamming

Slamming is the action of high waves raising the bow of a vessel on the sea surface, which causes physical damage to the ship's crew and system. It is linked to the ship's pitching and vertical movements and represents a significant seakeeping issue due to its potential for causing structural damage to the ship and discomfort to the crew. Investigating a vessel's response to wave-induced motions is crucial, and insights into this problem are gained through analysis of the ship's design and weather data. Denoting by L_{OA} the overall length of the ship, by d the ship draft, by v_{cr} the threshold velocity, by σ_0^2 variance in elevation, and by σ_2^2 the variance in velocity, the probability of bow slamming using Nielsen's (1987) assessment can be calculated as Equation (4).

$$s_B = P_{Slamming} = exp\left(-\left(\frac{v_{cr}^2}{2\sigma_2^2} + \frac{d^2}{2\sigma_0^2}\right)\right) \begin{cases} \leq 0.03 & for\ L_{OA} \leq 100m \\ \leq 0.01 & for\ L_{OA} > 100m \end{cases} \quad (4)$$

A detailed description of the computation of the aforementioned parameters is provided in Appendix C (Fathi, 2004). For instance, consider a vessel that has an overall length of 177 meters and a draft of 9.826 meters. The data associated with various ranges of significant wave height (H_s) and mean wave period of irregular waves (T_1) will be illustrated in Figure 7. For the implicit values of $H_s \geq 14.5$ the vessel would be threatened by bow slamming for the wide ranges of $6.5 \leq T_1 \leq 16.5$. However, the weather data with $H_s \leq 10.5$ for the same range as T_1 would be in a safe zone regarding bow slamming phenomenon. Figure 7 demonstrates the graph analysis of bow slamming probability for the given instance and shows the border limit of the safety of the vessel in connection with bow slamming.

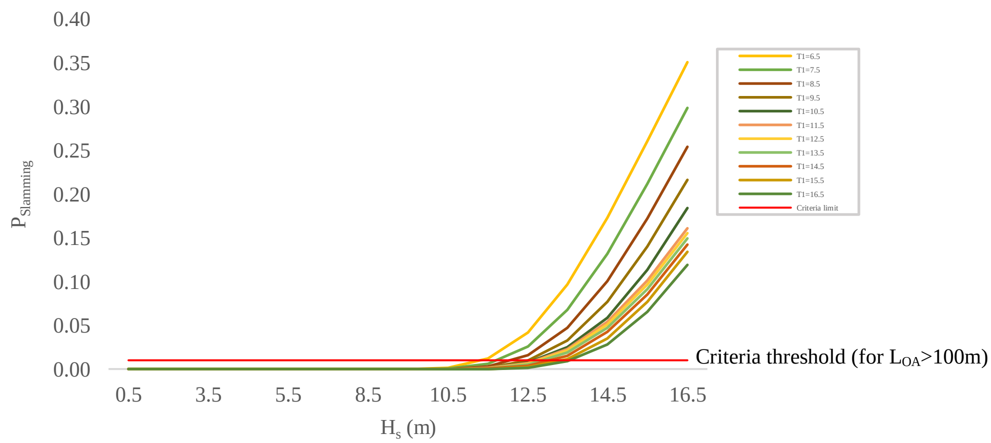

Figure 7. The probability graph of bow slamming for the given example.

3.3. Green water on deck

In rough weather, the ship motions of pitching and heaving can become excessively large, causing water to flow onto a vessel's deck. This phenomenon, known as green water on deck, threatens the ship's structure (i.e., deck plating, hatches), its cargo and crew. The probability of green water on deck is provided by Equation (5), which takes into account the vessel's freeboard (fb). The criteria used to evaluate the safety issue is based on NORDFORSK's assessment of ship performance in a seaway, as described by Nielsen (1987):

$$s_G = P_{Green\ water} = exp(-\frac{fb^2}{2\sigma_0^2}) \leq 0.07 \quad (5)$$

For example, suppose a vessel with a freeboard of 4.39 meters. If the significant wave height reaches 8 meters, the probability of green water rises to 0.071. This exceeds the safety threshold of 0.07, indicating a potential safety risk. However, in regions where the significant wave height is less than 7.98 meters, the vessel can safely navigate as the probability of green water remains within acceptable values.

In a vessel with 4.6 meters of freeboard, the probability of green water for different *Hs* is indicated in Figure 8. For the given example and $H_s \geq 8.5$, the green water probability would be utterly in the dangerous zone. To be more precise, according to the sensitivity analysis results illustrated in Figure 8, the vessel would not be allowed to sail in the ocean areas with $H_s \geq 7.98$ for safety purposes in the matter of green water loading concern.

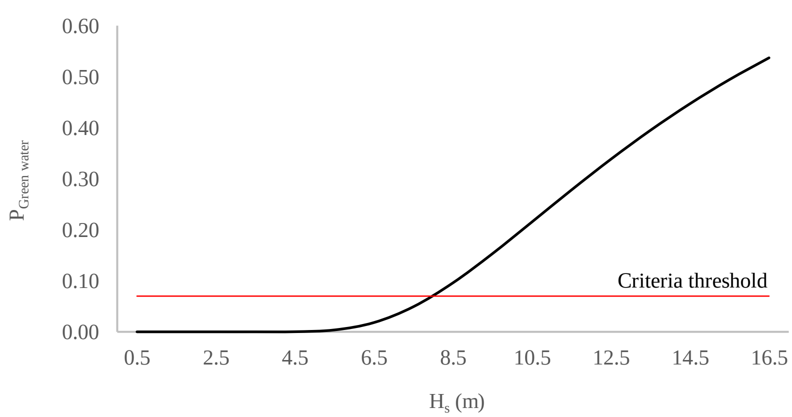

Figure 8. The probability graph of green water loading for the given example.

3.4. Other safety issues

Exploring the safety regulations specified in the IMO (2007) guideline reveals critical measures designed to protect vessels under various conditions. These regulations include the following essential issues: The surf-riding and broaching-to phenomena are situations associated with waves and ship speed. They may occur when the vessel's speed is the same or less than the wave's speed, causing the vessel to heel at a significant angle or suddenly change its heading. As a result, the vessel would be in danger of capsizing. This situation can arise when the angle between the vessel's heading and the direction of incoming waves falls between 135° and 225°, and the vessel speed exceeds a specific value (IMO, 2007). Successive high-wave attack refers to the situation where a vessel is encountered by a group of dominant waves traveling at the same speed. It would induce the danger of synchronous rolling motions, parametric rolling motions, intact stability reduction or combinations of various phenomena (IMO, 2007). This phenomenon happens when the average wave length exceeds 80% of the ship's length and the significant wave height is larger than 4% of ship's length. Synchronous rolling motion happens when the natural rolling period of a ship is the same as the encounter wave period in following and quartering seas. The synchronization between the ship's rolling period and the wave period can result in a back and forth rolling motion that can cause instability and increase the risk of capsizing. Parametric rolling motions may arise based on the vessel's position on the wave crest and wave trough, which puts the vessel in danger of a large heeling angle, subsequently increasing the risk of capsizing. It occurs when the encounter period equals or is half the rolling period. The aforementioned phenomena have been derived from the IMO (2007) guideline document. Depending on the related safety concern, these safety issues are affected by the vessel's speed, wave height, wave period, and the encounter angle between waves and the vessel's direction. When these safety aspects are combined, they represent both non-dangerous and dangerous zones that are limited by vessel and weather data. In the context of optimization, the terms “dangerous zone” and “non-dangerous zone” refer to the hard and soft constraints, respectively. Figure 9 provides a visual representation of the dangerous zones that planners needs to be aware of while navigating under certain weather conditions. This polar chart illustrates the speed and the relative angle to the waves (0° = head seas) for a specific ship, highlighting the areas at which potential threats to safety, as specified by the IMO guidelines, may be encountered.

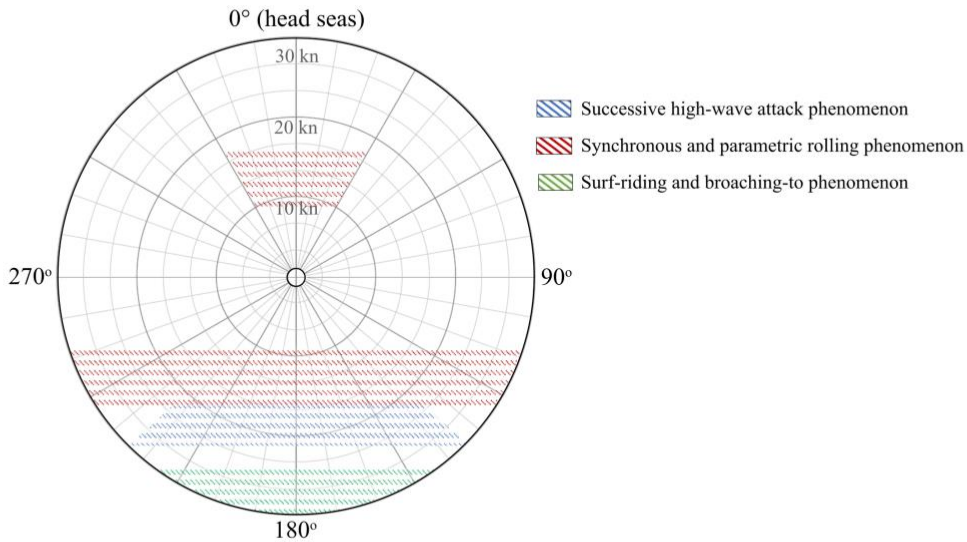

Figure 9. An illustration of dangerous zones for a specific ship, with different colors indicating various safety issues (successive high-wave attack in blue, synchronous and parametric rolling motions in red, and surf-riding and broaching-to in green) according to IMO guidelines, across different speeds and relative courses to waves.

The assessment of a vessel's seakeeping performance can be conducted using various methods, including the Response Amplitude Operator (RAO) and the Seakeeping Performance Index (SPI). RAO quantifies how the vessel responds to incoming waves during each of the six ship's motions. Specifically, RAO is a function of wave frequency and is illustrated as a ratio between the ship motion amplitude and the wave amplitude. Notably, the ship's motion is influenced by its speed, heading, and the wave frequency. Consequently, the RAO is formulated in relation to these parameters. High RAO values suggest that the ship is particularly sensitive to waves of certain frequencies, which could result in significant ship motions. Conversely, a ship with low RAO values responds more softly to incoming waves, leading to more stable and comfortable voyages at sea. However, there's a safety threshold for RAO configured to each hull design. Exceeding this threshold can compromise the vessel's safety.

SPI is derived from an aggregate measure that represents the vessel's overall seakeeping performance (Pennino et al., 2020). This value is computed using five reference criteria: the Root Mean Square (RMS) of pitch amplitude, RMS of vertical acceleration at the forward perpendicular, Motion Sickness Incidence (MSI), and the probabilities associated with slamming and green water occurrences. To formulate the SPI, each of the aforementioned reference criteria is first normalized against its respective safety threshold (Stevens and Parsons, 2002; Pipchenko and Zhukov, 2010) and subsequently subtracted from one. The SPI is then determined by selecting the maximum value between zero and the product of these normalized ratios. Each ratio serves as a comparative measure, contrasting the current state of the vessel with its established threshold for each criterion. It is important to highlight that if any criterion exceeds its safety threshold, the SPI defaults to a value of zero. The SPI's numerical range is consequently defined between zero and one. A low SPI value indicates that certain criteria are edging towards their safety thresholds, indicating a potential reduction in seakeeping performance. Conversely, an SPI value approaching one signifies higher seakeeping performance, reflecting reduced adverse ship motions.

4. Review of safety methodology

In the maritime industry, weather routing and route optimization are two closely related yet distinct approaches to navigating the seas. Weather routing is traditionally concerned with ensuring the safety and comfort by selecting a path that minimizes the impact of adverse weather conditions. This involves a precise analysis of weather forecasts and ocean conditions to navigate a path that avoids storms, high waves, or other hazards. On the other hand, route optimization takes a broader perspective, including weather considerations while also accounting for factors such as fuel efficiency, navigation time, and cost-effectiveness. Its goal is to find the most efficient path for a vessel, establishing a balance between timely arrivals, operational expenses, and environmental concerns. While weather routing is a subset of route optimization, focusing solely on meteorological aspects, its practical application has expanded to include various operational parameters in practice. This alignment with key maritime goals to enhance voyage performance while prioritizing safety. For a thorough and comprehensive analysis of weather routing methodologies and practices, we refer to the extensive study conducted by Zis et al. (2020).

To provide a background overview, twenty papers have been carefully selected based on their relevance to optimizing vessel routes under the soft and hard safety constraints, and their incorporation of multi-objective criteria aligned with practical maritime operational goals. These papers are organized by the specific safety issues they address, divided into four key areas: IMO guidelines, seakeeping criteria, dynamic stability, and other hazards.

4.1. IMO guidelines

Krata and Szlapczynska (2012) developed a MEWRA algorithm to find the optimized route where time, fuel consumption and safety index were minimized. The paper includes a set of hard safety constraints, including adherence to the IMO (2007) guidelines described before, avoidance of landmasses and shallow waters, and areas where wind speeds exceed 40 knots. To incorporate soft safety constraints, the authors introduces a safety index that takes into consideration both dangerous and non-dangerous zones, following the guidelines outlined in the IMO (2007) guidelines. This index accounts for surf-riding and broaching-to, and successive high-wave attacks. Accordingly, they define a safety index as the ratio of the non-dangerous area to the sum of both areas and use this index as an arc cost in the problem. In earlier work, Szlapczynska (2015) considered static (time-independent) constraints alongside dynamic (time-dependent) ones. The static constraints refer to the areas that must be excluded from the passage, such as landmasses and regions identified for piracy risks, which define the permanent navigational boundaries.

Meanwhile, the dynamic constraints are based on weather data, including high wind regions, which could change the navigation during the route planning period. Together, these constraints enable the adjustment of routes in response to evolving conditions, ensuring the optimization of routes within safe and efficient navigable spaces. In another work, Szlapczynska and Szlapczynski (2019) incorporate the DM's preferences early in the optimization process to determine a preference-based pareto frontier. Their method employs weight intervals for each objective to reflect the range of importance a DM might assign to them, rather than relying on fixed weights. At the start, the algorithm requires each objective to have a weight interval that reflects the DM's preference for that objective compared to others. In the solution selection phase of the MEWRA algorithm, a specific evaluation function is utilized to compare pairs of solutions across all objectives. This function assesses the relative performance of solutions based on how well they align with the DM's specified weight intervals. It relies on either the minimum or maximum weight from the intervals, depending on the performance comparison. This approach offers a customized optimization process, guiding the selection process towards solutions that better match the DM's preference spectrum.

Fabbri and Vicen-Bueno (2019) proposed a multi-criteria vessel routing problem that balances time, ship navigation added resistance, and the safety risk in relation to dangerous zones, based on the IMO (2007) guidelines. To address this problem, their study employs Martins' labeling algorithm as a solution methodology. The approach focuses on presenting a set of Pareto dominant solutions using a specialized visualization technique. This technique enables the effective representation of the Pareto frontier on a 2-D or 3-D graph, achieved by normalizing and aggregating multiple objective functions. The selection of specific objective functions to be grouped and aggregated is determined by the DMs. By doing so, DMs can visualize and analyze trade-off solutions in a more intuitive manner.

Veneti et al. (2017) presented a new labeling algorithm similar to Martin's labeling algorithm to minimize fuel consumption and safety risk in vessel route planning where travel time was constrained. The method estimates the travel time by a heuristic function, where the total duration of the voyage is limited. The safety formulation is applied as both soft and hard constraints. On the one hand, the hard constraints are applied according to IMO (2007) guidelines as described earlier, except that they only included parametric rolling, and surf-riding and broaching-to to form the dangerous zone. On the other hand, the soft constraints are formulated based on historical data, where the safety risk was calculated as the probability of accidents multiplied by the severity of their consequences for each arc within the network.

Li et al. (2017) employed an extended version of the GA, developed by Deb et al. (2002), which is specifically tailored for multi-objective optimization with a focus on finding the Pareto front. This approach incorporates advanced mechanisms, such as classifying solutions into different levels based on their dominance over others, and a method for maintaining diversity in the solution space, ensuring a comprehensive exploration and a diverse range of solutions. The objectives of their study include voyage duration, fuel consumption, and navigation safety risks. To quantify the navigation safety risk, they develop a criterion based on the maximum wave height that the vessel could safely navigate through.

4.2. Seakeeping criteria

In addressing the complexities of maritime routing problems, dynamic programming offers a methodical framework for optimization. Zaccone et al. (2018) applied this approach to specifically optimize fuel efficiency while maintaining the vessel's stability across all six degrees of motion. The optimization is subject to hard safety constraints ensuring that the ship's six motions remained within acceptable ranges. The evaluation of the six ship motions is accomplished through a series of mathematical equations and principles, which are grounded in physics and wave theory. The probability of specific motions exceeding thresholds is calculated using a concept known as the spectral moment of order zero. This concept quantifies the likelihood of the motion exceeding a particular magnitude, which enables assessing the potential discomfort caused by excessive movement. Spectral moment of order zero for each motion is computed using RAO.

Padhy et al. (2008) delved into the concept of RAO, employing Dijkstra’s algorithm to optimize travel time, taking vessel speed as a key decision variable. Their research implies that decreasing the ship's speed can enhance safety due to better seakeeping characteristics. The notion of safety is imposed as a hard constraint, monitored through the assessment analysis rooted in RAO. In their study, the RAO is computed for specific hulls, ship speeds, and headings. Then given the RAO's safety threshold, the decision regarding the speed variable is incorporated into the route optimization stage.

In the realm of maritime research, the impact of seakeeping performance on voyage length and route time has been a subject of exploration. Pennino et al. (2020) improved seakeeping performance by optimizing vessel routes to maximize the SPI, employing Dijkstra's algorithm. The findings suggest that ship seakeeping performance can be improved by as much as 50% without significantly increasing route length or voyage duration.

Zhao et al. (2021) introduced a combination algorithm that leverages the strengths of PSO and GA to address multi-conflicting objectives, such as fuel consumption, route time, and safety. In terms of safety considerations, the study took a three-dimensional approach, considering three factors: wind risk, wave risk, and SPI. The calculation of wind risk is based on IMO (2008) guidelines. This involves determining the maximum wind speed a ship can withstand and evaluating the ratio of the crosswind speed to this value to assess wind risk. To compute wave risk, a sophisticated methodology is applied, which focused on evaluating the ratio of the rolling period to the encounter period within three discrete intervals.

Depending on where this ratio falls within defined intervals, the assessment either indicates no significant risk or quantifies the wave risk based on the ratio. Their solution approach involves the utilization of an arithmetic crossover method, effectively merging two individuals to generate two new solutions, thus diversifying the solution space and enhancing exploration. For the local search phase, the researchers adopted a single-point Gaussian mutation technique, with the reference route serving as the mean value of the function. To identify the optimal Pareto solution set, the researchers employed a selection process based on nondominated sorting. This method allowed them to isolate the most promising solutions that offer the best trade-offs among the objectives.

Hinnenthal and Clauss (2010) introduced an approach to weather routing that employs a bi-objective GA to optimize ship routes under changing weather conditions. This method evaluates potential routes for their efficiency in terms of fuel consumption and Estimated Time of Arrival (ETA), while also taking into account weather forecasts to minimize exposure to adverse conditions. It further imposes hard safety constraints on bow slamming and vertical accelerations, significantly enhancing overall safety. The algorithm iteratively refines route options, balancing the trade-offs between different objectives, and adapts dynamically to changing weather patterns, leading to the identification of Pareto-optimal routes.

Vettor and Soares (2014) used an extended version algorithm of GA, developed by Zitzler et al. (2001), to address a multi-objective vessel routing problem. Their approach aims to identify the optimized route by leveraging the concept of the Pareto frontier. This algorithm evaluates both how many solutions are dominated and the distribution density of solutions. This ensures that the selected solutions are not only dominate but also diverse, avoiding over-concentration on a single solution type. The authors integrate objectives like path length, ETA, and safety in their study, with a particular focus on safety that included both hard and soft constraints. In this context, they specifically examine bow slamming probabilities as a key aspect of safety assessment.

Optimizing vessel routing in the presence of uncertain factors such as weather, sea conditions, and vessel performance has always been a complex endeavor. Vettor et al. (2020) developed a multi-objective metaheuristic approach, building on the algorithm they previously proposed in Vettor and Soares (2014). This approach utilizes probabilistic methods to assess seakeeping criteria, specifically addressing uncertainties associated with varying weather conditions. To predict these uncertainties, the paper applies a Gaussian probability density function. In the short term, the paper assesses hazardous situations to quantify safety risks associated with the arc cost of the network. For this purpose, the researchers compute the maximum allowable wave height and wave period based on the established seakeeping criteria. Employing an ensemble method, they then derive the safety risk function by evaluating the cumulative Gaussian probability function across wave height and wave period values that exceed the established thresholds. In the long run, weather routing is employed with a combined objective function that accounts for both time and safety risk. This objective function is formulated by multiplying the safety risk by the ratio of the time required to travel between neighboring nodes over the total voyage time. By combining these two factors, the proposed approach aims to optimize vessel routing, prioritizing both efficient travel time and safe navigation.

4.3. Dynamic stability

Krata and Szlapczynska (2018) presented another ship routing addressed by the MEWRA method to derive a set of Pareto optimal paths. The optimization aims to balance predefined objective weights, including fuel consumption, voyage duration, and safety. The hard safety constraints, as per the IMO (2007) guidelines, are accompanied by considerations of synchronous rolling, parametric rolling, surf riding, and broaching-to phenomena, serving to shape the solution space. Furthermore, the soft constraint, represented as arc costs, is defined based on the ratio of the current GM of the vessel to its initial GM. The initial GM indicates the vessel's maximum stability under calm water conditions and represents the maximum GM achievable. However, as the vessel navigates the open ocean, external forces like waves and winds cause a decline in the vessel's GM.

Sharif et al. (2024) proposed an inverse optimization approach to determine the objective weights in a multi-objective vessel routing problem that incorporates a weighted sum method. This study prioritizes key objectives such as fuel consumption, route time, and a variety of safety concerns, addressing both hard and soft constraints related to dynamic stability, green water on deck, and bow slamming. The methodology uses so-called best practice routes, generated from expert knowledge, weather data, and criteria analysis by domain experts. The approach is evaluated through the use of artificial best practice routes, which are generated by the optimization model with predefined objective weights applied.

Yang et al. (2022) combined the ACO with the TOPSIS (Lai et al., 1994) decision-making method. In this integrated approach, ACO generates potential routes, with each route evaluated for fuel efficiency and safety using TOPSIS. The safety risk in the paper is focused on the risk of a ship rolling due to strong winds and waves. This risk is quantitatively assessed based on the relative angles of wind and wave directions to the vessel. Therefore, the algorithm calculates the navigation safety risk by summing the combined risk due to wind and the risk due to waves, to assess the overall stability and safety of the vessel under challenging environmental conditions.

4.4. Other hazards

In military maritime context, naval mines are autonomous explosive devices strategically deployed to damage, deter, or destroy enemy ships and submarines. For maritime nations, such as South Africa, the United States of America, the United Kingdom, Japan, China, Russia, and Australia, the presence of naval mines poses significant risks, threatening the safety of both military and civilian vessels navigating near conflict zones, historical minefields, or strategic maritime routes. In maritime routing problems, situations involving minefields present a significant challenge, as these areas must be excluded from the solution space. Bekker and Schmid (2006) devised a solution to overcome this issue by combining the Dijkstra's and GA algorithms. Their goal was to find the safest and shortest path between origin and destination points, taking into account uncertainties or variations in mine performance, detection accuracy, and environmental conditions that could affect safety margins. To achieve this, they used a weighted objective function with two key objectives: trajectory risk and voyage length, where the DM assigned weights to each objective based on their judgment. The risk function, crucial for calculating the relative danger of routes that pass near the edge of no-go zones, is formulated by a ratio based on the inverse value of the total distances to all the mines. The GA's purpose is to selectively remove the minimum number of mines needed to create a safe passage through a known minefield while minimizing the route length. In this case, each gene in the individual represents the presence or removal of a mine. The fitness function evaluates each candidate solution based on the number of mines removed and the resulting route length. The best solution provides the optimum arrangement of removed mines for a safe passage.

To address the challenges of navigating through areas with numerous obstacles, Yanchin and Petrov (2019) developed a bi-objective approach. Their method employs an evolutionary algorithm and a parallel GA to optimize vessel route planning for both the safest and shortest path. To address the safety hard constraint, this paper considers the ship's draught, which is the minimum depth required for safe passage, ensuring that the planned route avoids areas with depths less than its draught. Additionally, the paper acknowledges the possible impact of water currents and tides on depths, but it simplifies the analysis by assuming constant depths and disregarding temporal variations. The study employs a two-step algorithm for optimal route planning. The first algorithm shares similarities with traditional GA but without mutations and crossover. It evaluates the fitness of each point based on its proximity to the destination, the ability to move forward, and avoid obstacles. In the second phase, each entire route plotted in the first phase is treated as an individual within parallel GA. This phase runs multiple GAs in parallel, each with distinct configurations, to enhance efficiency and diversity. The focus is on determining the safest and fastest path, with a fitness function that emphasizes quicker arrival times at the destination. Upon completion of the algorithm, the Pareto front is obtained.

Navigating the challenges of maritime safety, various studies have proposed innovative solutions. One notable contribution comes from Kuhlemann and Tierney (2020) who applied a GA to optimize routes by balancing fuel costs and safety concerns, particularly with avoiding pirate zones. The algorithm, while emphasizing fuel efficiency, penalizes routes that experience delays or enter into hazardous zones. Routes are labeled safe or unsafe using a binary variable, depending on their passage through danger zones. In the proposed algorithm, the authors present an initial solution that directly connects the starting point to the destination. They then iteratively refine the route, adjusting midpoints between land and water orthogonally to prevent intersections with the land. To introduce route variations, the researchers present two crossover techniques. The first merges routes by pinpointing a common midpoint, while the second randomly pairs a point from one route's first half with another from the second route's latter half. For mutations, they outline a strategy where one of nine operations is chosen at random. The operations range from deleting or moving points, to making simultaneous adjustments, or even modifying a route based on wind conditions, turning radius constraints, or vessel speed adjustments.

Vessel route planning in cold waters presents unique challenges due to ice navigation. Liu et al. (2016) addressed this issue by developing a GIS-based tool to support ice navigation and finding optimal routes for ships in icy conditions. The objectives of their research includes minimizing voyage length and maximizing the distance to unnavigable ice regimes. The GIS-based solution is carried out in two stages. In the first stage, the researchers divide the sea into navigable and unnavigable areas based on the computation of the Ice Numeral (IN), a polynomial function that indicates the navigability of ice regimes.

The IN takes into account various factors such as ice types, ice concentrations, thickness, and vessel type. Positive values of the IN represent navigable sea areas, while negative values indicate unnavigable regions. To determine the constant coefficient of the polynomial, the researchers utilize an Ice Multiplier table. Moving to the second stage, the researchers discretize the space of the navigable area into individual lines that vessels could navigate in. To achieve this, they employed a Voronoi Diagram which is a partitioning-based method that divides a plane into a finite number of regions based on proximity to a set of objectives, known as generators or seeds. In the above research, the generators are defined as obstacles representing non-navigable areas, helping the algorithm to keep vessels at a safe distance from high ice concentration regions. By utilizing Dijkstra's algorithm in conjunction with the GIS-based tool and the Voronoi Diagram approach, this paper provides a practical solution for ice navigation.

Ari et al. (2013) solved a single-objective shortest path problem using the A* algorithm. Their work takes into account the safety hard constraints, formulated as a buffer zone around obstacles, which a ship must not enter. The boundary of this region consists of points that are a specified distance away from the closest point on the obstacle's boundary. These safety concerns serve two main purposes: ensuring a safe distance from hazards like debris, rock formations, small islands, ice blocks, other ships, or coastlines, and providing a margin for navigational inaccuracies that might occur due to environmental conditions or course deviations. The model developed in the study computes optimal navigation routes that adhere to these safety constraints, while also taking into account factors such as the ship's turning radius and speed changes during maneuvers.

Table 1 presents a summary of the methodologies and safety considerations addressed in the reviewed papers. In the realm of maritime routing, various safety considerations are seamlessly integrated into the decision-making processes, alongside other critical factors like fuel efficiency and voyage duration. These criteria are either applied as constraints or as objective functions, depending on the context. Specifically, safety criteria were applied as either soft or hard constraints in the studies reviewed. For instance, when an obstacle is present, the immediate priority is to maintain a safe distance, typically achieved by removing the associated arcs in the network. However, it is vital to note that this is just one dimension of the broader safety spectrum. Other vital dimensions encompass standards set by the IMO, seakeeping criteria, and previously identified safety challenges in Section 3. Regrettably, a comprehensive model that simultaneously embraces all these safety issues, while also optimizing for fuel consumption and route time, has yet to be thoroughly explored. This underscores a distinct research gap: there's a significant need for in-depth studies that incorporate every aspect of safety in vessel routing, alongside an optimization approach tailored for problems with many objectives. This would ensure comprehensive protection for cargo, vessel infrastructure, and onboard crew or passengers.

Table 1. A summary of the reviewed papers in vessel routing problems coupled with safety considerations.

Reference	Safety hard constraints				Safety soft constraints via objective function									Other objectives					Solution method	Weight selection
	IMO guidelines	Seakeeping	Dynamic stability	Other hazards	IMO guidelines	Seakeeping	Dynamic stability	Obstacles	Wind risk	Wave risk	Bow slamming	Green water on deck	Accident risk	Fuel	Time	Distance	ETA	Added resistance		
Sharif et al. (2024)	X	X	X				X				X	X		X	X				Simplex	Inverse optimization
Yang et al. (2022)									X	X				X					ACO	DM
Zhao et al. (2021)						X			X	X				X	X				PSO and GA	DM
Kuhlemann and Tierney (2020)								X						X					GA	DM
Pennino et al. (2020)						X													Dijkstra	-
Vettor et al. (2020)						X								X	X				GA	DM
Fabbri and Vicen-Bueno (2019)	X			X	X										X			X	Martins' labeling	DM
Szlapczynska and Szlapczynski (2019)	X			X	X									X	X				MEWRA	DM
Yanchin and Petrov (2019)				X				X							X				Parallel GA	DM
Krata and Szlapczynska (2018)	X						X							X	X				MEWRA	DM
Zaccone et al. (2018)		X												X					Dynamic programming	-
Li et al. (2017)										X				X	X				GA	DM
Veneti et al. (2017)	X												X	X					Martins' labeling	DM
Liu et al. (2016)				X				X								X			Dijkstra	DM
Szlapczynska (2015)	X			X	X									X	X				MEWRA	DM
Vettor and Soares (2014)											X					X	X		GA	DM
Ari et al. (2013)				X									X		X				A*	-
Krata and Szlapczynska (2012)	X			X	X									X	X				MEWRA	DM
Hinnenthal and Clauss (2010)		X				X					X			X			X		GA	DM
Padhy (2008)		X													X				Dijkstra	-
Bekker and Schmid (2006)									X							X			Dijkstra and GA	DM

5. Concluding remarks

This paper contributes to advancing knowledge in maritime navigation and safety through a detailed analysis of current challenges and solutions in maritime routing. Our work encompasses the exploration of multi-objective planning, safety considerations in routing, and innovations in route optimization with a focus on safety. Collectively, these contributions underscore the critical role of multi-objective programming and safety in enhancing maritime routing strategies.

Before tackling optimization problems, especially in scenarios involving weighted sum multi-objective optimization, it is crucial to accurately estimate the weights of different objectives to prioritize them effectively. This step encompasses both subjective and objective aspects of weight estimation methods. Subjective decision-making, encompassing tools like point allocation, direct rating, and ranking techniques, offers flexibility but faces limitations such as potential bias and lack of precision. In contrast, objective methods, represented by the AHP and the entropy weighting system, rely on measurable data and structured mathematical models to mitigate subjective biases. However, methods like entropy assume a direct correlation between data behavior and importance, while AHP often struggles with challenges related to time consumption and complexity, particularly in scenarios involving numerous criteria for comparison. The study highlights a research gap in developing reliable objective weights that are both statistically valid and aligned with the practical complexities of decision-making scenarios. This underscores the need for multi-criteria decision analysis methods that balance subjective and objective factors effectively in real-world scenarios. Although inverse optimization integrates both aspects, its efficiency should be validated using real best practice routes determined by real-world weather data over short-term planning periods. Addressing this gap necessitates a large-scale network model and extensive historical data, including weather forecasts.

Once objective weights are established, the focus shifts to solving the optimization problems. This phase employs various algorithms and heuristics designed to navigate through the complexities of routing networks and multi-objective planning, seeking to find optimal or near-optimal solutions. Algorithms like Dijkstra's and A* are foundational for identifying efficient paths within networks by minimizing cost, including route time, fuel consumption and safety metrics. Pareto optimality is also highlighted to identify balanced solutions. We demonstrate the adaptive routing strategies through heuristic methods like MEWRA, GA, PSO, and ACO. These methods generate diverse solution sets that approximate the Pareto front, offering decision-makers a spectrum of optimal trade-offs between conflicting objectives.

The imperative of navigating adverse weather conditions and ensuring safety cannot be overstated. The safety of a vessel is multi-dimensional, including IMO guidelines, seakeeping criteria, dynamic stability and obstacle avoidance. We have introduced a specific formulation designed to integrate critical safety aspects into the decision-making process, including the probability of bow slamming, the occurrence of green water, and dynamic stability.

The maritime industry's increasing reliance on voyage optimization highlights a significant shift towards multi-dimensional problem-solving approaches, integrating factors like time, fuel consumption, and safety. This integration is crucial for achieving efficient, cost-effective, and safe overseas voyages. Recent advancements in the development of sophisticated routing algorithms play a pivotal role in enhancing maritime navigation. These frameworks balance environmental-related objectives by considering weather conditions and vessel characteristics. Various optimization techniques, including MEWRA, GA, Dijkstra’s, and A* algorithms, illustrate the breadth of strategies employed to navigate the multifaceted challenges of maritime routing. A notable aspect of this research landscape is the emphasis on DM preferences, highlighting a trend towards customizable and adaptive optimization processes. This collective body of work illuminates the complexity of maritime routing, showcasing the endeavor to balance safety with efficiency in pursuit of sustainable and responsible navigation practices in the maritime sector. However, challenges persist in the improvement of multi-objective optimization techniques, especially when addressing the varied dimensions of safety issues simultaneously, adding a layer of complexity to achieving the ideal balance in maritime routing solutions.

The practice of scaling objectives to fit within the same interval is fundamental to ensure that no single objective disproportionately influences the routing outcome due to its larger numerical values. This normalization process is crucial for maintaining the integrity of the optimization process, ensuring that all objectives are given equal consideration regardless of their original scales. Yet, this task is made more complex by the variability and potential inaccuracies in weather data. Given the unpredictable nature of marine conditions, routing decisions must account for the possibility that current data may not remain accurate, necessitating adaptable strategies that can respond to updated information. Further complicating the optimization is the integration of expert knowledge in setting objective weights. This knowledge is crucial for aligning routing decisions with real-world operational preferences and insights. However, the challenge lies in accurately converting this subjective expertise into numerical weights that genuinely reflect the experts' intentions. The weights initially set may misrepresent the desired impact on routing decisions. Addressing these challenges requires a comprehensive approach that combines mathematical models with practical insights and experience, alongside the flexibility to adapt to changing conditions.

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Declarations of interest

None.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT only to improve the English grammar and sentence structure.

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Appendix A

Using the notation described below, the formulas for calculating KM and KG are as follows (Sharma, 2015; Kristensen and Lützen, 2012; Adam, 2018; Schneekluth and Bertram, 1998; Kupras, 1976).

Description	Symbol	Unit
Basic vessel dimension (input)		
Length between perpendicular	L_{PP}	m
Length between waterline	L_W	m
Main engine power	P_B	HP
Depth	D	m
Breath	B	m
Draught	d	m
Calculated vessel particulars		
Volume displacement	∇_{volume}	m^3
Waterplane coefficient	C_{WP}	-
Midship section coefficient	C_M	-
Block coefficient	C_{B1}	-

$$\nabla_{\mathrm{volume}} = \nabla\ 0.99108963072$$

$$\mathrm{C_{B1}} = \frac{\nabla_{\mathrm{volume}}}{(L_{PP}\ \mathrm{B\ d})}$$

$$\mathrm{C_{WP}} = 0.55 + 0.45\mathrm{C_{B1}}$$

$$\mathrm{C_M} = 0.977 + 0.085(\mathrm{C_{B1}} - 0.60)$$

$$\mathrm{W_{light\ ship}} = \mathrm{W_{steel}} + \mathrm{W_{outfit}} + \mathrm{W_{engine}}$$

$$\mathrm{W_{steel}} = \mathrm{W_{hull}} + \mathrm{W_{forecastle}} + \mathrm{W_{deckhouse}}$$

$$\mathrm{W_{hull}} = 3.28\ \mathrm{c\ d\ z^{0.69} L} \left(1.104 - 0.016\frac{\mathrm{L}}{\mathrm{B}}\right).\left(0.53 + 0.04\frac{\mathrm{L}}{\mathrm{D}}\right).\left(1.98 - 0.04\frac{\mathrm{L}}{\mathrm{D}}\right).\left(1.146 - 0.0163\frac{\mathrm{L}}{\mathrm{D}}\right)$$

$$\text{Factor c} = 1 + \frac{0.73}{\sqrt{\mathrm{L}}}$$

$$\text{Factor d} = \text{Weight reduction factor ,}$$

If ship's hull is constructed from mild steel: d=1

If ship's hull is constructed from higher tensile steel: $d = \left(1 - \frac{l_{KF}(1-RKF)}{L\, f_4}\right)$

Where:

$RKF = \frac{51}{\delta_F + 27}$ where δ_F is steel's yield strength in kg/mm^2

l_{KF} is the equivalent length within cargo spaces, where higher tension steel is used.

Correction factor $f_4 = 1.146 - 0.0163 \frac{L}{D}$

$$\text{Factor } z = 2.1FBL^2 \frac{(C_{B1} + 0.7)}{10^6}$$

Factor F

$L < 240 \quad F = 3.0408175 + 0.014826515\, L - 0.0000173469\, L^2$

$240 \leq L \leq 300 \quad F = 1.320 + 0.0298333\, L - 0.00005\, L^2$

$L > 300 \quad F = 5.77$

$W_{forecastle} = 0.014LB$

$W_{deckhouse} = 160 + 0.00874LB$

$W_{outfit} = 277 + 0.115LB$

$$W_{engine} = P_B \frac{895 - 0.0025\, P_B}{10^4}$$

$$KG_{steel} = 0.01D\left(46.6 + 0.135\,(0.81 - C_B)\left(\frac{L}{D}\right)^2\right) + 0.008\,D\left(\frac{L}{B} - 6.5\right)$$

with correction factor if:

ship has bulb bow, $\quad - 0.002D$

ship's length <= 120m, $\quad + \left(1 - \frac{L-60}{60}\right) 0.001D$

$KG_{outfit} =$

$D + 1.25 \quad , \text{if } L \leq 125$

$D + 1.25 + 0.01(L - 125)\,, \text{ if } 125 < L < 250$

$D + 2.50 \quad , \text{if } L \geq 250$

$KG_{engine} = 0.17d + 0.36D$

$$KG = \frac{W_{steel}\, KG_{steel} + W_{outfit}\, KG_{outfit} + W_{engine}\, KG_{engine}}{W_{light\, ship}}$$

$$KM = \left\{\begin{array}{ll} B\left(13.61 - 45.4\left(\frac{C_{B1}}{C_{WP}}\right) + 52.17\left(\frac{C_{B1}}{C_{WP}}\right)^2 - 19.88\left(\frac{C_{B1}}{C_{WP}}\right)^3\right) & ,0.73 < \frac{C_{B1}}{C_{WP}} < 0.95\} \\ B\left(0.08\ C\ \frac{\frac{B}{d}}{\sqrt{C_M})} + \frac{0.9-0.3C_M-0.1C_{B1}}{\frac{B}{d}}\right) & , else. \end{array}\right\}$$

Appendix B

The wind heeling levers l_{w1} and l_{w2} are constant values at all angles of inclination and should be calculated as follows (DNV GL, 2019):

$$l_{w1} = \frac{P \,.\, A \,.\, Z}{1000\, g.\, \nabla} \quad \text{(B.1)}$$

$$l_{w2} = 1.5\, l_{w1} \quad \text{(B.2)}$$

It this formulation, P refers to the wind pressure, while A is the lateral area of the ship and deck cargo above the waterline. The parameter Z describes the vertical distance from the center of lateral resistance to about half the draft or the underwater lateral center. The term center of lateral resistance designates a point on a ship's hull. At this point, the combined lateral force generated by the water pressure acting sideways on the hull is believed to be concentrated. Meanwhile, g stands for gravitational acceleration, and ∇ represents the ship's displacement.

The angle of roll θ_1 in degrees is calculated according to the methodology outlined by DNV GL (2019), where the values for the factors X_1, X_2, k, and s are obtained from Table B.1 to Table B.4, respectively.

$$\theta_1 = 109kX_1X_2\sqrt{rs} \qquad \text{(B.3)}$$

The value of the parameter r is computed based on OG, which is the distance between the center of gravity and the waterline; it is assigned a positive (+) value if the center of gravity is below the waterline, and a negative (−) value if it is above.

$$r = 0.73 \pm 0.6\frac{OG}{d} \qquad \text{(B.4)}$$

The parameter s is determined based on the rolling period, which is calculated as follows:

$$T = \frac{2C_{B2}}{\sqrt{GM}} \qquad \text{(B.5)}$$

The value of the parameter X_2 depends on the block coefficient (C_{B2}), which itself is a function of the breadth to depth ratio (*B/d*) and the length of the ship at the waterline (L_{wl}). The parameter A_k represents the total overall area of bilge keels, the area of the lateral projection of the bar keel, or the sum of these areas.

$$C_{B2} = 0.373 + 0.023\left(\frac{B}{d}\right) - 0.043\left(\frac{L_{wl}}{100}\right) \qquad \text{(B.6)}$$

Table B.1. Values of factor X_1, adapted from DNV GL, 2019.

B/d	X_1
≤2.4	1
2.5	0.98
2.6	0.96
2.7	0.95
2.8	0.93
2.9	0.91
3	0.9
3.1	0.88
3.2	0.86
3.3	0.84
3.4	0.82
≥3.5	0.8

Table B.2. Values of factor X_2, adapted from DNV GL, 2019.

C_B	X_2
≤0.45	0.75
0.5	0.82
0.55	0.89
0.6	0.95
0.65	0.97
≥0.7	1

Table B.3. Values of factor *k*, adapted from DNV GL, 2019.

$\mathbf{A_k 100}/\mathbf{L_{wl} B}$	k
0	1.05
1	0.98
1.5	0.95
2	0.88
2.5	0.79
3	0.74
3.5	0.72
≥4	0.7

Table B.4. Values of factor *s*, adapted from DNV GL, 2019.

T	s
<6	0.1
7	0.098
8	0.093
12	0.065
14	0.053
16	0.044
18	0.038
≥20	0.035

To compute wind pressure, there is a need to calculate relative wind direction (Ψ_{wr}) based on wind direction (Ψ_{wt}) and the vessel's heading (Ψ_{s}). Accordingly, relative wind speed (v_{wr}) is formulated using wind speed (v_{wt}) and vessel speed (v_{s}). The values mentioned above can be found using the law of cosines:

$$\overrightarrow{v_{wt}} = \overrightarrow{v_s} + \overrightarrow{v_{wr}} \qquad \text{(B.7)}$$

$$v_{wr} = \sqrt{v_{wt}^2 + v_s^2 + 2 v_{wt} v_s \cos(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}})} \qquad \text{(B.8)}$$

$$\Psi_{\mathrm{WR}} = \begin{cases} \tan^{-1}\left(\frac{v_{wt} \sin(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}})}{v_s + v_{wt} \cos(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}})}\right) & , v_s + v_{wt} \cos(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}}) \geq 0 \\ \tan^{-1}\left(\frac{v_{wt} \sin(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}})}{v_s + v_{wt} \cos(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}})}\right) + 180 & , v_s + v_{wt} \cos(\Psi_{\mathrm{wt}} - \Psi_{\mathrm{s}}) < 0 \end{cases} \qquad \text{(B.9)}$$

Wind pressure is computed using the mean added resistance due to wind (ITTC, 2014) divided by the area of the maximum transverse section exposed to the wind.

$$R_{AA} = \frac{1}{2}\ \rho_A\ v_{wr}^2\ C_X(\Psi_{\mathrm{WR}})\ A_F \qquad \text{(B.10)}$$

$$P = \frac{R_{AA}}{A_F} \qquad \text{(B.11)}$$

where

$\mathrm{R_{AA}}$ is the mean added resistance due to wind

$\mathrm{A_F}$ is the area of maximum transverse section exposed to the wind

$\mathrm{C_X}$ is the wind resistance coefficient

$\mathrm{\rho_A}$ is the Mass density of air

The wind load coefficient C_X is crucial for calculating the wind forces acting on a ship, taking into account ψ_{wr} and coefficients such as C_{LF}, C_{XLI}, and C_{ALF}. These coefficients reflect the influence of the ship's superstructure shapes and configurations on wind force dynamics. The determination of these coefficients is grounded in empirical research, notably the study conducted by Kitamura et al. (2017).

$$C_{X(\psi_{WR})} = \begin{cases} C_{LF}.\cos(\psi_{WR}) + C_{XLI}.\sin(\psi_{WR}).\cos(\psi_{WR})\left(\sin(\psi_{WR}) - \frac{\sin(\psi_{WR}).\cos(\psi_{WR})^2}{2}\right) + C_{ALF}.\sin(\psi_{WR}).\cos(\psi_{WR})^3 & ,\psi_{WR} \neq 90 \\ C_{X(90)} = \frac{C_{X(80)}+C_{X(100)}}{2} & ,\psi_{WR} = 90 \end{cases} \qquad \text{(B.12)}$$

The above water structural of a vessel is an essential parameter to determine the wind loads on ship as is extensively described in the research done by Kitamura et al. (2017) using a regression function. This estimation depends on vessel length and breadth. The following sections provides the detailed methodology.

$$\left.\begin{matrix} A_L \\ C \\ H_C \\ A_{OB} \\ A_F \\ H_{BR} \end{matrix}\right\} = a.B + b.L_{OA} + C \qquad \text{(B.13)}$$

where

A_{OD} is the lateral projected area of superstructures etc. on deck

A_L is the projected lateral area above the waterline

C_{MC} is the horizontal distance from midship section to center of lateral projected area A_{YV}

H_{BR} is the height of top of superstructure (bridge etc.)

H_C is the height from waterline to center of lateral projected area A_{YV}

Table B.5 presents the regression coefficients necessary for calculating the aforementioned parameters.

Subsequently, the parameters C_{LF}, C_{XLI}, and C_{ALF} are computed using the data provided in Table B.6.

$$C_{LF} = \begin{cases} \beta_{10} + \frac{\beta_{11}.A_L}{L_{OA}.B} + \frac{\beta_{12}.C}{L_{OA}} & 0 \leq \psi_{WR} < 90 \\ \beta_{20} + \frac{\beta_{21}.B}{L_{OA}} + \frac{\beta_{22}.H_C}{L_{OA}} + \frac{\beta_{23}.A_{OD}}{L_{OA}} + \frac{\beta_{24}.A_F}{B^2} & 90 < \psi_{WR} \leq 180 \end{cases} \tag{B.14}$$

$$C_{XLI} = \begin{cases} \delta_{10} + \frac{\delta_{11}.A_L}{L_{OA}.H_{BR}} + \frac{\delta_{12}.A_F}{B.H_{BR}} & 0 \leq \psi_{WR} < 90 \\ \delta_{20} + \frac{\delta_{21}.A_L}{L_{OA}.H_{BR}} + \frac{\delta_{22}.A_F}{A_L} + \frac{\delta_{23}.B}{L_{OA}} + \frac{\delta_{24}.A_F}{B.H_{BR}} & 90 < \psi_{WR} \leq 180 \end{cases} \tag{B.15}$$

$$C_{ALF} = \begin{cases} \varepsilon_{10} + \frac{\varepsilon_{11}.A_{OD}}{A_L} + \frac{\varepsilon_{12}.B}{L_{OA}} & 0 \leq \psi_{WR} < 90 \\ \delta_{20} + \frac{\varepsilon_{21}.A_{OD}}{A_L} & 90 < \psi_{WR} \leq 180 \end{cases} \tag{B.16}$$

Table B.5. The regression intercept and coefficient to determine above water structural shape of a vessel. Adapted from Kitamura et al., (2017).

		Ship Type								
		Tanker (ballast)	Tanker (full)	Bulker (ballast)	Bulker (full)	LNG (ballast)	LNG (full)	Container (full)	Passenger	Others
A_L	a	0.00152	0.00044	86.63000	69.44000	1.15900	0.94720	0.27620	-0.01352	-2.74500
	b	-0.00027	-0.00020	4.66500	-3.46600	-0.09198	-0.06462	0.03156	0.00376	1.01700
	c	0.07559	0.07791	-602.70000	559.30000	0.93360	0.51850	2.71800	0.52850	1.56600
C	a	0.66270	1.13100	0.00000	0.27930	0.25660	-0.19590	0.15290	-0.60520	0.02854
	b	-0.13200	-0.24790	0.02314	-0.10670	-0.04442	0.02552	0.04705	0.00000	0.00000
	c	0.13860	1.33000	-9.52100	3.46200	-1.85500	-1.72600	-19.32000	11.52000	-0.04393
H_C	a	0.00000	-0.17040	0.16170	0.11220	0.81720	0.73490	0.18530	0.23340	0.14370
	b	0.03099	0.04311	-0.00966	-0.00933	-0.07892	-0.07028	0.00906	0.03650	0.03011
	c	1.76600	2.63700	5.96200	5.45900	0.22410	0.38230	2.71100	-0.88950	0.46100
A_{OD}	a	-21.72000	-21.72000	27.61000	27.61000	0.00000	0.00000	0.00000	0.00000	-0.24890
	b	5.48000	5.48000	0.00000	0.00000	-0.00359	-0.00359	12.42000	0.00229	0.06123
	c	-33.03000	-33.03000	-129.60000	-129.60000	2.74000	2.74000	-944.60000	0.03658	0.98610
A_F	a	0.00000	0.00000	17.92000	26.06000	1.13200	1.01800	0.00000	-0.02765	0.51270
	b	-0.00003	-0.00003	1.14000	-2.44700	-0.06409	-0.05437	0.08992	0.00518	0.08065
	c	0.02571	0.02094	-75.15000	218.30000	4.22100	4.20300	8.50000	0.82590	-0.63990
H_{BR}	a	-0.16840	-0.00092	-0.41040	0.08064	1.42200	1.30600	0.18910	-0.02008	0.40480
	b	0.12030	0.00000	0.07818	-0.03038	-0.08359	-0.07310	0.07587	0.00351	0.08309
	c	6.66300	0.13350	19.42000	21.13000	2.40700	2.35400	7.19600	0.92660	0.80920

Table B.6. The coefficients used to compute C_{LF}, C_{XLI} and C_{ALF}.

Parameter		j				
		0	1	2	3	4
β_{ij}	i = 1	0.922	-0.507	-1.162	-	-
	i = 2	-0.018	5.091	-10.367	3.011	0.341
δ_{ij}	i = 1	-0.458	-3.245	2.313	-	-
	i = 2	1.901	-12.727	-24.407	40.31	5.481
ε_{ij}	i = 1	0.585	0.906	-3.239	-	-
	i = 2	0.314	1.117	-	-	-

Appendix C

Denoting by H_S the significant wave height, by γ the peakedness parameter (Fathi, 2004), by T_1 the mean wave period of irregular waves, and by T_P and ω_P the peak period and frequency of wave spectrum, respectively, we provide a detailed description of the parameters required to calculate the bow slamming probability.

$$v_{cr} = 0.093\sqrt{gL_{OA}} \tag{C.1}$$

$$\sigma_0^2 = m_0 = \frac{1}{16}H_S^2 \tag{C.2}$$

$$\sigma_2^2 = m_2 = \frac{1}{16}H_S^2\omega_P^2\frac{11+\gamma}{5+\gamma} \tag{C.3}$$

$$\omega_P = \frac{2\pi}{T_P} \tag{C.4}$$

$$T_P = 1.296\,T_1 \tag{C.5}$$

$$\gamma = \begin{cases} 5 & , \frac{T_P}{\sqrt{H_S}} \leq 3.6, \\ exp(5.75 - 1.15\frac{T_P}{\sqrt{H_S}}) & , 3.6 < \frac{T_P}{\sqrt{H_S}} < 5, \\ 1 & , 5 \leq \frac{T_P}{\sqrt{H_S}}\ . \end{cases} \tag{C.6}$$